Examination of Combined Parameters of Lightning Discharges and Atmospheric Conductivity Necessary for Generation of Red Sprites

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Red sprites are the most important transient luminous events in the lower ionosphere above thunderstorms, whose physical mechanisms and role in the Earth's atmosphere are not yet fully understood. According a popular hypothesis, they are driven by quasi-electrostatic fields (QESF), which are created after a lightning discharge and cause electric breakdown and generate streamers in the region 50-90 km. The goal of this paper is to examine the set of parameters by which QESF generated are strong enough for sprite occurrence. We also explain why sprites sometimes occur after discharges with quite small charge moment change (CMC) (CMC<<1000 C×km), while in other cases they are not realized even after very powerful lightning discharges (CMC>1000 C×km). For this goal we use a model of QESF proposed by us in earlier works. Our results show that for the occurrence of a sprite a combination of conditions revealed in the paper have to be fulfilled which concern parameters of both the lightning discharge and the atmospheric conductivity.

Introduction

Red sprite is the most frequently observed phenomenon among several different types of transient luminous events in the middle atmosphere above thunderstorms, which have been discovered during the last 10-15 years [1]. Sprites occur at night a short time (up to tens of milliseconds) after a positive cloud-to-ground (CG+) lightning discharge [1, 2]. A typical sprite comprises the altitudinal region between 50 and 90 km. It is a process in the time scale from milliseconds up to few tens of milliseconds. Since the sprite onset is not earlier than several milliseconds after the causative lightning, it is suggested that sprites are driven by strong post-lightning quasi-electrostatic fields (QESF) which appear in the mesosphere and lower ionosphere in a proper timescale, instead of by the electromagnetic pulse generated in a shorter time-scale. The QESF are formed due to the temporal unbalance of the spatial electric charges after a lightning discharge; these fields exist until a new charge balance is reached [2-4]. According to the theoretical investigations [2-5] these electric fields can be strong enough to cause an electrical breakdown at mesospheric altitudes and to create positive and negative streamers.

The role of red sprites is not fully understood yet. Presumably they influence the global atmospheric electrical circuit, as well as the atmospheric chemistry balance. Also, they can be dangerous for space craft launches. Numerous experimental campaigns [6,7] and theoretical investigations have been devoted to the exploration of sprites, in order to explain their physical mechanisms and their role. A series of observations demonstrate that in their lower part (<75 km) sprites usually have a filamentary structure [8]. In agreement with these observations a sprite is represented by a three level structure [2]: up to ~75 km the sprite is realized as a network of streamers; above 85 km a region of diffuse glow is formed; between these both there is a transition region.

Although many results are obtained, the sprite physics is still not completely explained. One of the questions to be answered is why sprites usually occur above mesoscale convective systems, and appear much more rarely above single thunderstorm cells producing very powerful discharges. An explanation can be that the sprites are typically produced by CG+ lightning discharges accompanied with continuing currents, and not by ordinary lightning [9]. The difference between both types is schematically illustrated in Fig.1: although of the same maximal amplitude, the sprite-producing lightning discharge is characterized by a long tail due to continuing currents [7], which can be easily formed in the mesoscale convective systems. A question remains why the CG+ discharges of this type produce sprites easier.

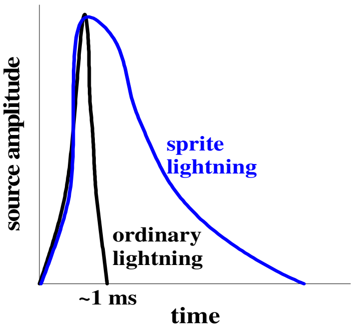

Fig.1. Comparison of waveforms of sprite-producing and non-sprite-producing lightning.

Besides, although a large charge moment change (CMC > 600 - 1000 C×km) of the CG+ lightning discharge is usually needed for a sprite production, sometimes sprites occur after very week lightning discharges (CMC of few hundreds of C×km). In other cases, even strong CG+ discharges (CMC>1000 C×km) do not cause sprites.

To explain these peculiarities we consider the occurrence of a sprite as a combined function of the lightning discharge parameters and the atmospheric conductivity parameters. Our study is based on the popular hypothesis that sprites are realized by an electrical breakdown realized by post-lightning quasi-electrostatic fields; in their lower part sprites are realized as a network of streamers. The post-lightning QESF by different parameters are evaluated by our model [3].

Such investigation is important for better explanation of the physics of sprites and their parameters.

Study of post-lightning QESF as function of lightning and atmospheric parameters

Model of QESF

Our model for the special and temporal distribution of the QESF **E** generated after a CG+ lightning discharge [4] is based on the continuity equation for the Maxwell current density $\mathbf{j}_M = \sigma\mathbf{E} + \varepsilon_0\, \partial\mathbf{E}/\partial t$, where ε_0 is the dielectric constant, σ is the atmospheric conductivity as a function of altitude z:

$$\mathrm{div}\, j_M = 0 \qquad (1)$$

Assumptions are used that: (***i***) **E** is a potential field; (***ii***) the atmospheric conductivity is isotropic in the model domain (0 - 100 km); (***iii***) the lightning discharge is described by a negative exponential function of the decay of the total charge *Q*0 at altitude *Z*Q to be removed: the remaining charge at time *t* after lightning onset is $Q(t) = Q_0 \exp(-t/\tau_L)$, by discharge characteristic time τL. Eq.(1) is solved by the following initial and boundary conditions [3]: ***1***) The initial electric field E0 at the beginning of the lightning discharge (at t=0) is created by constant initial charge *Q*0 located at altitude *Z*Q; ***2***) At time *t*: (***a***) the electric potential is zero at lower (z=0 km) and upper (z=100 km) boundaries; (***b***) $\mathrm{div}\,\mathbf{E} = Q(t)/\varepsilon_0$ over a thin layer at $z=Z_Q$.

Study of QESF which produce breakdown and streamers

First an example is considered of the QESF generated at different altitudes above a CG+ lightning discharge as a function of time *t* (Fig.2). A normalized charge of Q_0=1 C located at Z_Q=10 km is destroyed in characteristic time τ_L=1 millisecond. The example demonstrates the typical features of the QESF [3, 5]. In particular, QESF at any altitude z reaches its peak $E_P(z)$ at time *t* after the discharge onset: $t \approx \tau_R$ where τ_R is the local free charge's relaxation time. Another important characteristic is the time period $\Delta t_P(z)$ in which the QESF intensity $E(z)$ at altitude z is large (close to its peak E_P, i.e. $E(z) > E_P(z)/e$). At altitudes z with relaxation time $\tau_R < \tau_L$ this time period is $\Delta t_P(z) \approx \tau_L$ (it does not depend on the altitude). At lower altitudes z, where $\tau_R > \tau_L$, $\Delta t_P(z)$ increases with the height decrease, and is approximately equal to τ_R.

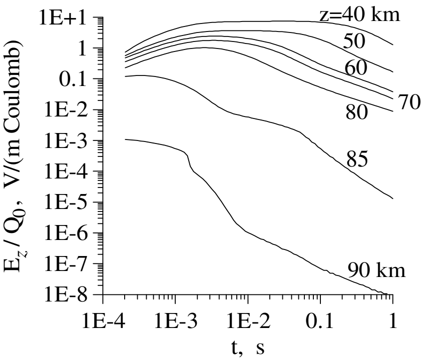

Fig.2. QESF at altitudes 40-90 km as function of time by discharge parameters Z_Q=10 km, τ_L=1 ms.

E_P and Δt_P are the key characteristics which determine the sprite realization via post-lightning QESF [3, 5]. A breakdown is realized at an altitude z when $E_P(z) > E_B(z)$. Here E_B is the breakdown electric field (minimal intensity sufficient to realize breakdown): $E_B(z) = C_B N(z)/N(0)$, where $N(z)$ is the atmospheric neutral density at altitude z, $C_B = 3.2\times10^6$ V/m is the breakdown electric field at the sea level [2]. The similar condition is valid for generation of a negative and a positive streamers, as well, but by constants C_{SN}=1250 kV/m, C_{SP}=440 kV/m, respectively, instead of C_B.

The ability of an electrical breakdown at an altitude z depends on the parameters of the causative lightning. Fig.3 demonstrates the minimal CMC (CMC_0) of the discharge which is sufficient for a breakdown at altitude z (CMC_0 is a function of z) by lightning parameters Z_Q=10 km, τ_L=1 ms. Two cases considered are for a nighttime and for a daytime conductivity profile, respectively. Fig.3 demonstrates that sprites are possible only at night. The magnitude Q_B of the removed charge, which is sufficient for a breakdown, reaches its minimum at the altitude Z_R where the charge's relaxation time τ_R is equal to the discharge time τ_L [4].

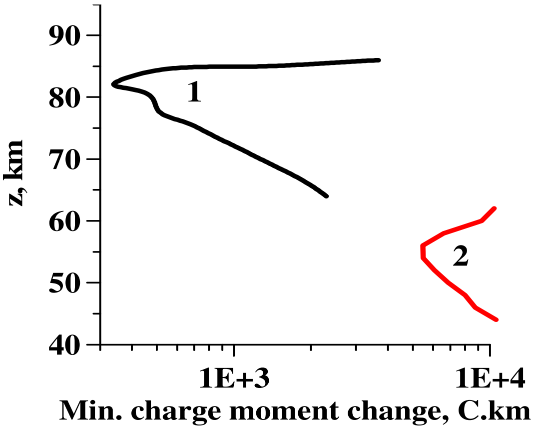

Fig.3. Minimal CMC needed for a breakdown at altitude z by night (curve 1) and daytime (2). The required CMC is smallest at z where the relaxation time tR is equal to discharge time tL.

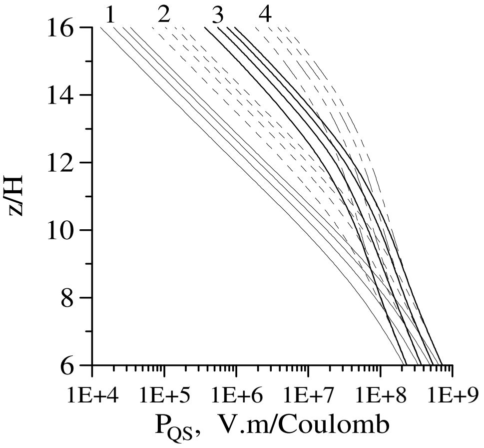

Fig.4. $P_{QS}(\zeta)$ depending on z_Q , z_L. Nrs 1-4 correspond to z_L=8,10,12,14. Curves in each group are for z_Q=1, 1.5, 2, 2.5.

Conditions of sprite realization

The occurrence of a sprite depends on the interplay between the scale-heights H_P and H_B of the altitudinal dependences of the peak QESF E_P, and of the breakdown electric field E_B, respectively [4]. H_B is equal to the well known scale-height of the atmospheric neutral density. We study the scale-height H_P as a combined function of parameters of the lightning discharge and of conductivity.

Study of peak QESF as a combined function of parameters

First we study the profile of the peak QESF intensity $E_P(z)$ as a combined function of the lightning discharge parameters Z_Q, Q_0 and τ_L and the conductivity parameters by assumption for an exponential conductivity profile σ_E. Then we generalize these results for realistic conductivity profiles.

In an idealistic case that $\sigma(z)=\sigma_E=\sigma_0\exp(z/H_C)$ (σ_0 is the atmospheric conductivity at surface, H_C is the conductivity scale-height) we represent the profile of the normalized peak of QESF $P_{QS}(\zeta)=H_C^2E_P/Q_0$, where $\zeta=z/H_C$, as a common function of the dimensionless parameters of the lightning discharge $z_Q=Z_Q/H_C$ and $z_L=Z_R/H_C$ which is an indirect discharge time parameter. This dependency is illustrated in Fig.4, where profiles P_{QS} are computed by different sets of parameters. Three general features are demonstrated: 1) Each profile has a knee at height Z_K where $\tau_R=\tau_L$; 2) The scale height H_{PE} of any profile is much smaller (≈1) well above the knee Z_K, than well below it ($H_E\approx 2.7$); 3) P_{QS} decreases with the increase of the discharge time τ_L, but only above the knee Z_K; it becomes insensitive to τ_L below the knee.

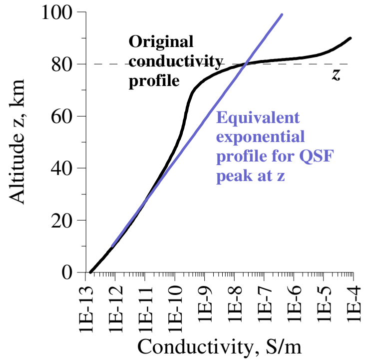

Fig.5. Illustration of the exponential conductivity profile equivalent to the original with respect to QESF peak at z=80 km.

We show, by comparative analysis in different cases, that the peak QESF $E_P(z)$ obtained by a realistic conductivity profile σ can be approximated by the value $Q_0P_{QS}(z/H_C)/H_C^2$ obtained for an exponential profile σ_E which coincides with σ at altitude z and at the charge's height Z_Q [4]. A sample conductivity profile and its approximation by an exponential profile, which is used to evaluate $E_P(z)$ is demonstrated in Fig.5. With respect to such approximation we apply the results for P_{QS} obtained by exponential conductivity profiles in cases of a realistic nighttime conductivity profile (this profile can be preliminary modified due to electron heating above the thunderstorm [3]). An important feature of the profiles of this type is the presence of an expressed knee at an altitude Z_{KC} (70-90 km for different conductivity profiles) of a transition from ion to electron conductivity). The knee of two sample nighttime conductivity profiles is illustrated in Fig.6.

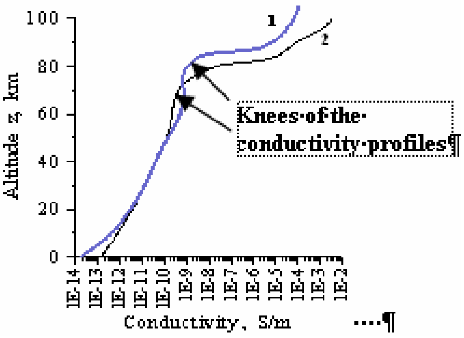

Fig.6. Two nighttime conductivity profiles: 1- according to []; 2 - from [].

For a case of a CG+ lightning discharge considered above we transform the generalized functional dependence for P_{QS} demonstrated in Fig.4 with respect to each of these conductivity profiles [4], and by substitution the dimensionless characteristics to their physical analogues. As a result, the height profiles of E_P shown in Fig.7 (curves 1 and 2) are obtained. Two knees typically appear in these E_P profiles: one at altitude $z=Z_{KR}$ with relaxation time $\tau_R=\tau_L$, and another $z=Z_{KC}$ which corresponds to the knee of the conductivity profile. Below the lower knee the scale-height H_{PL} of the E_P profile is bigger than the scale-height H_B of the breakdown electric field E_B; above the upper knee H_{PU} is smaller than H_B; between both knees its value H_{PM} is intermediate. Therefore, both knees determine the height interval where the breakdown or the streamer formation is easier. According to an estimation Z_{KC} varies between 75 and 90 km depending on latitude and various conditions [10-15]. Z_{KR} depends on the conductivity profile, and also on the discharge time τ_L. Typically τ_L~1 ms (in case of an ordinary CG+ lightning, see Fig.1), but is several times larger when continuing currents are involved.

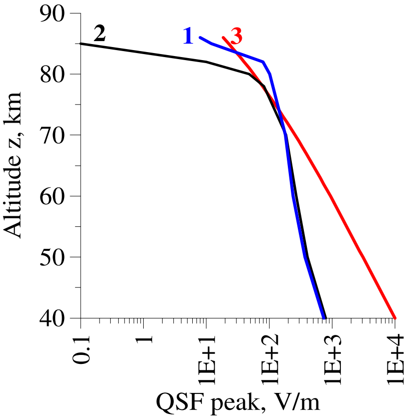

Fig.7. QESF peak by a +CG discharge with CMC=1000 C.km, τ_L=1 ms. Curves 1, 2 correspond to conductivity profiles 1, 2 in Fig.6; 3 shows the breakdown field.

For conductivity model [10] Z_{KR}= 82 km and 81 km, respectively by τ_L=1 and 5 ms. According to conductivity model [12] Z_{KR}=78 and 76 km; according to [13-15] Z_{KR}=72 and 71 km by τ_L=1 and 5 ms.

The role of the conductivity profile in sprite realization is demonstrated by Figures 6, 7. In Fig.6 two different conductivity profiles ([10] and [12]) are demonstrated. Fig.7 shows two profiles of the QESF peak computed for each conductivity profile, and are compared with the profile of the breakdown electric field (curve 3). Fig.7 shows that E_P significantly depends on the conductivity only above Z_{KR}, and is not sensitive to it at lower altitudes.

The breakdown produced by the minimum CMC necessary is realized at one of the knees Z_{KR} and Z_{KC} or within the interval between both knees. With the CMC increase the height interval of breakdown realization becomes wider. Above both knees Z_{KR}, Z_{KC} the scale-height H_P of the QESF peak E_P is very small (from 0.7 km up to few km [4]): this means that the upper boundary Z_U of the sprite region can be only up to several kilometers above both knees. The sprite lower boundary Z_L is estimated from the average conductivity scale-height H_{CL} between the charge's height Z_Q and the lower knee:

Conditions necessary for generation of observable sprites

Streamers are developed in the lower portion of a sprite, where the relaxation time τ_R is larger than both the dissociative attachment time τ_a and the time τ_s of the development of individual electron avalanche into a streamer [2]. Thus, the transition between the streamer region and the region of diffuse glow approximately coincides with the height interval between both knees Z_{KR} and Z_{KC}. This conclusion is confirmed: our estimations of Z_{KR} and Z_{KC} is in agreement with the height structure of sprites determined from observations.

The temporal and spatial characteristics of post-lightning QESF are in agreement with those of a sprite. According the observations, the time periods of luminosity in the different sprite's parts approximately coincide with the time periods Δt_P of big electric field intensity at the respective altitudes. Indeed, in its upper part the glowing starts earlier and prolongs much shorter time (less than 1 ms) than in its lower (streamer) part [2]. This demonstrates that sprites are well observable when their streamer portions are spread to lower altitudes (50 km or lower), where Δt_P is larger. A well developed streamer section is typical in sprites, since the time-scale of large QESF at altitudes below the knee Z_{KR} allows formation and propagation of positive and negative streamers.

Thus, it is required for a sprite occurrence that $E_P > E_B$ well below (tens of kilometers) both knees Z_{KR} and Z_{KC}. We showed above that the peak QESF E_P in the region below Z_{KR} does not depend on the discharge time τ_L and is proportional to the CMC of the lightning discharge. Since the total charge transported from a thunderstorm cell to the ground is much larger in a case of a CG+ discharge with continuing currents than by an ordinary CG+ lightning (Fig.1), the peak QESF E_P in the streamer region will be much larger in the first case than in the second case. This means that a sprite are created much easier after CG+ lightning discharges with continuing currents than after ordinary discharges, which can be an explanation of the fact that sprites occur much easier above mesoscale convective systems.

The sprites generated by weak lightning CG+ discharges possibly occur due to significant modifications of the stratospheric conductivity. This will be a subject of our next investigation.

Conclusions

- The profile of the quasi-electrostatic field peak has two knees (typically in the interval 70-85 km): one located at the knee of conductivity profile and another at the equalization of the relaxation and the discharge times. A sprite is initiated closely to one of these knees.
- QESF is large in a time-scale of streamer formation and propagation or larger at altitudes with relaxation time bigger than the discharge time.
- The sprite occurrence significantly depends on the +CG charge moment change and on the average scale-height of the atmospheric conductivity. The sprite is influenced by the discharge time only in its upper part of diffuse glow.
- Large continuing currents by a +CG lightning discharge possibly are the main factor for generation of streamers below ~75km;
- Lightning discharges with large continuing currents are better candidates for sprites, than the ordinary discharges, due to stronger QESF in the streamer region at about 75 km and below in a timescale relevant for streamer formation.

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