Determination of Total Mechanical Energy of the Universe within the Framework of Newtonian Mechanics

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The recent astronomical observations indicate that the expanding universe having a finite particle horizon is homogeneous, isotropic and asymptotically flat. The Euclidean geometry of the universe enables to determine the total kinetic and gravitational energies of the universe within the framework of Newtonian mechanics. It has been shown that almost the entire kinetic energy of the universe ensues from the cosmological expansion. Both, the total kinetic and gravitational energies of the universe have been determined in relation to an observer at arbitrary location. It is amazing that the modulus of the total gravitational energy differs from the total kinetic energy with a multiplier close to a unit. Thus, the total mechanical energy of the universe has been found close to zero. Both, the total kinetic energy and the modulus of total gravitational energy of the universe are estimated to 3/10 of its total rest energy Mc^2.

Introduction

The problem for the total average density of the universe $\overline{\rho}$ acquires significance when it has been shown that the General Relativity allows revealing the large-scale structure and evolution of the universe by simple cosmological models [1-3]. Crucial for the geometry of the universe appears dimensionless total density $\Omega = \overline{\rho} / \rho_c$, where ρ_c is the critical density of the universe. The most trustworthy total matter density Ω has been determined by measurements of the dependence of the anisotropy of the Cosmic Microwave Background (CMB) upon the angular scale. The recent results show that $\Omega \approx 1 \pm \Delta\Omega$, where the error $\Delta\Omega$ decreases from 0.10 [4, 5] to 0.02 [6]. The fact that Ω is so close to a unit is not accidental since only at $\Omega = 1$ the geometry of the universe is flat (Euclidean) and the flat universe was predicted from the inflationary theory [7]. The total density Ω includes densities of baryon matter $\Omega_b \approx 0.05$, cold dark matter $\Omega_c \approx 0.25$ [8] and dark energy $\Omega_\Lambda \approx 0.70$ producing an accelerating expansion of the universe [9, 10].

The found negligible *CMB* anisotropy $\frac{\delta T}{T} \sim 10^{-5}$ indicates that the early universe was very homogeneous and isotropic [11]. Three-dimensional maps of the distribution of galaxies corroborate homogeneous and isotropic universe on large scales greater than 100 *Mps* [12, 13].

Usually, Einstein pseudotensor is used for determination of the total energy of the universe [14, 15]. This approach is general for open, close and flat anisotropic models, but pseudotensorial calculations are dangerous as they are very coordinate dependent and thus, they may lead to ambiguous results [16]. Euclidean geometry of the universe still enables unambiguous determining of the total kinetic and gravitational energies of the universe, within the framework of classical mechanics.

Determination of total gravitational energy of the universe

The finite time of cosmological expansion H^{-1} (age of the universe) and the finite speed of light c set a finite particle horizon $R \sim cH^{-1}$ beyond which no material signals reach the observer. Therefore, for an observer in arbitrary location, the universe appears a 3-dimensional, homogeneous and isotropic sphere having finite radius (particle horizon) close to Hubble distance $R \sim cH^{-1}$. The possible matter beyond the particle horizon does not affect the observer.

The gravitational potential energy U of a homogeneous sphere having density $\overline{\rho}$ is:

$$U = -G\int_0^R \frac{M(r)dm}{r} = -\frac{16}{3}G\pi^2\overline{\rho}^2\int_0^R r^4 dr = -\frac{3}{5}\frac{GM^2}{R} \qquad (1)$$

where G is the gravitational constant, $M(r) = \frac{4}{3}\pi r^3\overline{\rho}$, $M = M(R)$ is the mass and R is radius of the observable universe.

The finite radius (particle horizon) of the universe is close to Hubble distance $R \sim cH^{-1}$, where $H \approx 70\ km\ s^{-1}\ Mps^{-1}$ [17] is the Hubble expansion rate and $H^{-1} \approx 1.4\times 10^{10}$ *years* is the Hubble time (age of the universe).

In consideration of $R \sim cH^{-1}$ and $\overline{\rho} = \Omega\rho_c$, we obtain for the mass of the universe:

$$M = \frac{4}{3}\pi R^3\overline{\rho} = \frac{4}{3}\frac{\pi c^3\Omega\rho_c}{H^3} \qquad (2)$$

The critical density of the universe [18] is determined from equation (3):

$$\rho_c = \frac{3H^2}{8\pi G} \qquad (3)$$

Replacing (3) in (2) we have found equation (4) for the mass of the universe including dark energy:

$$M = \frac{c^3\Omega}{2GH} \qquad (4)$$

Finally, replacing (4) in (1) and in consideration of $R \sim cH^{-1}$ we have found for the total gravitational energy of the universe:

$$U = -\frac{3}{20}\frac{c^5}{GH}\Omega^2 \qquad (5)$$

Similar approach has been used for calculation of the total gravitational energy of a body arising from gravitational interaction of the body with all masses of the observable universe [19, 20].

Determination of total kinetic and mechanical energies of the universe

The estimation of the total kinetic energy of the universe T is complicated as a result of the diversity of movements of masses in the universe. We suggest that almost all kinetic energy of the universe is a result of the cosmological expansion since it includes movement of the enormous masses (galaxies and clusters of galaxies) with average speed of the order of magnitude of $c/2$. The rotation curves of galaxies show that the majority of stars move into the galaxies with speed less than $v_0 = 3\times10^5$ *m/s* [21]. Besides, on rare occasions, the peculiar (non-cosmological) velocities of galaxies exceed this value [22]. On the other hand, the speed of medium-distanced galaxies (and their stars), as a result of the cosmological expansion is of the order of magnitude of $c/2 = 1.5\times10^8$ *m/s*. Obviously, the kinetic energy of an "average star" in the universe, ensuing from its peculiar movement, constitutes less than $(2v_0/c)^2 \sim 4\times10^{-6}$ part of its kinetic energy, ensuing from the cosmological expansion, therefore, former should be ignored.

Let us estimate the total kinetic energy of the universe in relation to an observer at arbitrary location. The total kinetic energy of the universe is the sum of the kinetic energy of all masses m_i moving in relation to the observer with speed v_i determined from Hubble law $v_i = Hr_i$, where $r_i \le cH^{-1}$ is the distance between the observer and mass m_i placed within the particle horizon:

$$T = \frac{1}{2}\sum_i m_i v_i^2 \qquad (6)$$

Although distant galaxies recede from the observer with speeds v_i comparable to c, the Newtonian formula (6) of kinetic energy is used. Thus, all calculations are made within the framework of the Newtonian mechanics.

Since for an arbitrary observer the universe appears a three-dimensional, homogeneous and isotropic sphere having finite radius (particle horizon) $R \sim cH^{-1}$, the sum (6) can be replaced by integral:

$$T = \frac{1}{2}\int mv^2 dr = \frac{1}{2}\int_0^R 4\pi r^2 \rho v_r^2 dr = 2\pi\bar{\rho}\int_0^R v_r^2 r^2 dr \qquad (7)$$

Replacing v_r with expression from Hubble law $v_r = Hr$, equation (7) is transformed into:

$$T = 2\pi\bar{\rho}H^2\int_0^R r^4 dr = \frac{2}{5}\pi\bar{\rho}H^2R^5 \qquad (8)$$

In consideration of $R \sim cH^{-1}$ we obtain:

$$T = \frac{2}{5}\frac{\pi\bar{\rho}c^5}{H^3} \qquad (9)$$

Since $\bar{\rho} = \Omega\rho_c$ (9) is transformed into:

$$T = \frac{2}{5}\frac{\pi\Omega\rho_c c^5}{H^3} \qquad (10)$$

After replacing (3) in (10) we have found the equation for the total kinetic energy of the universe:

$$T = \frac{3}{20}\frac{c^5}{GH}\Omega \qquad (11)$$

It is amazing that the equations for the total gravitational and kinetic energies of the universe (5) and (11) differ one from another by a multiplier $\Omega \approx 1$.

Clearly, the total mechanical energy of the universe E is:

$$E = T + U = \frac{3}{20}\frac{c^5}{GH}\Omega(1-\Omega) \qquad (12)$$

Since the resent *CMB* measurements obtain $\Omega \approx 1$, then $E = T + U \approx 0$, i.e. the total mechanical energy of the universe is close to zero. This substantial result confirms the conjecture that the gravitational energy of the universe is approximately balanced with its kinetic energy of the expansion [23].

Formulae (5) and (11) show the total kinetic energy of the universe is close to the modulus of its total gravitational energy:

$$T = |U| \approx \frac{3}{20}\frac{c^5}{GH} \qquad (13)$$

From Einstein equation and (4) we can estimate the total rest energy of the universe:

$$E_0 = Mc^2 = \frac{1}{2}\frac{c^5}{GH}\Omega \approx \frac{1}{2}\frac{c^5}{GH} \qquad (14)$$

Therefore, the total kinetic energy of the universe is 3/10 of its total rest energy E_0. The same is valid for the modulus of the total gravitational energy of the universe, too.

Discussions

Formulae (13) and (14) show that the total gravitational energy of the universe is $U = -\frac{3}{10}E_0 = -\frac{3}{10}Mc^2$, i. e. 30% of the total rest energy of the universe. On the other hand, the gravitational energy of a sphere having mass m and radius r equal to its Schwarzschild radius $r_S = \frac{2Gm}{c^2}$ is:

$$U = -\frac{3}{5}\frac{Gm^2}{r_S} = -\frac{3}{10}mc^2 \qquad (15)$$

which also appears 30% of the total rest energy of the mass m.

This suggests that the particle horizon ("radius") of the universe (R) coincides with Schwarzschild radius of the universe (R_S). It is easy to prove this statement using the following:

On account of $\Omega = \bar{\rho}/\rho_c \approx 1$ and $H \sim c/R$ we obtain:

$$\Omega = \frac{\bar{\rho}}{\rho_c} = \frac{3M/4\pi R^3}{3H^2/8\pi G} = \frac{2GM}{R^3H^2} \sim \frac{2MG}{Rc^2} \approx 1 \qquad (16)$$

Therefore, $R \sim c/H \approx \frac{2GM}{c^2} \equiv R_S$, i. e. the Schwarzschild radius of the universe coincides with particle horizon (radius) of the universe.

The formulae for total mass and rest energy of the universe (4) and (14) would be found by totally different approach, namely dimensional analysis, without consideration of the total density of the universe. In fact, a quantity with mass dimension $M = kc^{\alpha}G^{\beta}H^{\gamma}$ could be composed by using the fundamental parameters speed of the light (c), gravitational constant (G) and Hubble constant (H). Dimensional analysis gave α = 3 and β = γ= -1. Therefore:

$$M = k\frac{c^3}{GH} \sim 10^{53}\ kg \quad (17)$$

where $k \sim 1$.

Analogously, a quantity with dimension of energy $E = k_0 c^a G^b H^d$ could be composed by parameters c, G and H. In this case dimensional analysis has shown that a = 5 and b = d = -1. Therefore,

$$E_0 = k_0\frac{c^5}{GH} \sim 10^{70}\ J \quad (18)$$

where $k_0 \sim 1$.

Obviously, the formulae (17) and (18) coincide with formulae of total mass and rest energy of the universe (4 and 14), with accuracy to a factor k = 1/2. The total mass and rest energy of the universe have been found by original independent approach, namely dimensional analysis, which reinforces our presumptions for accepted cosmological model.

Conclusions

The recent astronomical observations indicate that the expanding universe having a finite particle horizon is homogeneous, isotropic and flat. The Euclidean geometry of the universe enables to determine the total kinetic and gravitational energies of the universe within the framework of the Newtonian mechanics.

Based on the simple homogeneous and isotropic model of the flat universe which expands according to Hubble law, we have found equations (5) and (11) for the total gravitational and kinetic energy of the universe, respectively. It is amazing that these equations differ by a multiplier $\Omega \approx 1$. That is, the modulus of the total gravitational energy of the universe is close to its total kinetic energy and therefore the total mechanical energy E of the universe is close to zero, i. e. the gravitational energy of the universe is balanced with its kinetic energy of the expansion.

Both, the total kinetic and gravitational energies of the universe have been determined in relation to an observer at arbitrary location. The total kinetic energy and the modulus of the total gravitational energy of the universe are estimated to 3/10 of its total rest energy Mc^2.

Finally, it has been shown that Schwarzschild radius of the observable universe coincides with particle horizon (radius) of the universe. Besides, the total mass and rest energy of the universe have been estimated by dimensional analysis, without consideration of total density of the universe.

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