New Orbital Distances Algorithm in Planetary Systems: The Moons of Uranus

Velinov P.I.Y., Yuskolov D.

Solar-Terrestrial Influences Institute, Bulgarian Academy of Sciences, Sofia, Bulgaria

The Titius-Bode law, which is generalized by Velinov and Yuskolov (2009) for the planets in Solar system, is applied for Uranian satellites also. New algorithm for the distances and fixed places of the satellites in Uranian system is proposed. We suggest linear quantization of space in the gravitational field of Uranus on the base of the exponential function $\tilde{d}_i = 2^{x-2} + C$ *, as C = 1 for every i = 1, 2, 3,..., 21 (i = 6 for Cordelia; i = 7 for Ophelia; i = 8 for Miranda and Ariel; i = 9 for Umbriel, i = 10 for Titania and Oberon, etc.). The function* $\tilde{d}_i$ *multiplied with the mathematical constant* π *forms the interpolating linear functional* $l_i(\tilde{f})$*, which contains information about the distance of the i-th satellite to Uranus. This functional is nonlinear because of the nonlinear dependence between the distance* r_i *from Uranus and orbital period* T_i *according to the second Kepler law. Some satellites have the same quantum number i because of the short distances between their orbits, i.e. the orbits have almost equal major semiaxis. Some quantum numbers i remain not fulfilled, which means that there are yet not discovered satellites. The places where they are situated are predicted in this paper.*

Introduction

Uranus, the seventh planet of the Solar System, has 27 known natural satellites with diameters ranging in size from 11 km to 1,578 km [1-5].

Listed in order of increasing orbital distance from Uranus, they are: Cordelia, Ophelia, Bianca, Cressida, Desdemona, Juliet, Portia, Rosalind, Cupid, Belinda, Perdita, Puck, Mab, Miranda, Ariel, Umbriel, Titania, Oberon, Francisco, Caliban, Stephano, Trinculo, Sycorax, Margaret, Prospero, Setebos, and Ferdinand [6-9].

In the study of Velinov and Yuskolov [10] a new generalized algorithm is proposed for the Titius-Bode law. As is well known the Titius-Bode law is an empirical formula, which we tried to improve with the corresponding linear quantization of space. This generalized Titius-Bode law [10] for the planets in the Solar system, was applied successfully for Jovian and Neptunian satellites in [11] and [12], respectively. The goal of the present work is the further enlargement of these applications, namely by means of quantitative consideration of the distances of the satellites in the Uranian system.

This problem is not yet solved for the moons in the Solar system [13]. In the paper of Panov [14] are considered all 27 satellites of Uranus. The results for the thirteen inner moons and for the five major moons are comparatively good (the errors are less than 7%). But the errors increase in the last most distant group (the nine irregular moons) until 23%. Panov [14] remarks for the system of Uranus that "it is not possible to find a single exponential formula for all Uranian satellites... Data for all 27 known satellites of Uranus are listed in Table 4b". That is why the problem with radial distances of Uranian satellites remains yet open.

New algorithm for the distances and fixed places of the Uranian satellites

We suggest linear quantization of space in the gravitational field of Uranus on the base of the exponential function

$$\tilde{d}_i = 2^{x-2} + \text{C}, \text{ as C} = 1$$

for every i = 1, 2, 3,..., 21 (i = 6 for Cordelia; i = 7 for Ophelia; i = 8 for Miranda and Ariel; i = 9 for Umbriel, i=10 for Titania and Oberon, etc.). The function $\tilde{d}_i$ multiplied with the mathematical constant π forms the interpolating linear functional $l_i(\tilde{f})$, which contains information about the distance of the i-th satellite to Uranus. This functional is nonlinear because of the nonlinear dependence between the distance r_i from Uranus and orbital period T_i according to the second Kepler law. After this law the radius vector $|\mathbf{r}| = r_i$ from Uranus to the given satellite describes equal surfaces for equal time intervals.

In our calculation we use several magnitudes and constants related to the seventh planet in our Solar system. The following values are very often used in the calculations in Tables 1 and 2:

1) The mass of Uranus is 1/19 314 from the mass of the Sun [2]. Or, we can write:

$$\text{M}_\text{U} = 4.373 \times 10^{-5}\,\text{M}_\text{S}$$

2) The square root from the Uranian mass in units M_S = 1 is:

$$(\text{M}_\text{U})^{1/2} = 6.613 \times 10^{-3}$$

3) The product of π with the Uranus mass is:

$$\pi\,\text{M}_\text{U} = 1.374 \times 10^{-4}\,\text{M}_\text{S}$$

For the Uranian satellite system the general case of the second Kepler's law is valid which is characterized by the product

$$T_i \times \sqrt{x \neq 1} \qquad (1)$$

where T_i is the orbital period of the i - th satellite (in years), x = $\text{M}_\text{U} \times (\text{M}_\text{S} = 1)$ is the mass of the central body, in our case - the planet Uranus, which mass is expressed in units of solar mass (M_S = 1). The product (1) is divisor in the right side of the equation

$$l_i(\tilde{f}) = \left(\frac{\pi\, r_i^2}{T_i \,.\sqrt{x \neq 1}} \right)^2$$

in the case $x < 1$. r_i is the module (the magnitude) of the radius - vector (i.e. the distance) from i-th satellite until the planet Uranus, which is expressed in AU (1 AU=149 597 870 km).

The square root from the mass of the central body when $x \neq 1$ changes the period of the tour of the satellite around Uranus. As was noticed for the satellite system of Uranus $x < 1$ and the period decreases.

The mathematical model of the second Kepler's law for the Solar system ($x = 1$) is [10]:

$$\left(\frac{\pi\, r_i^2}{T_i\,.\sqrt{x=1}}\right)^1 = \frac{\pi\, r_i^2}{T_i} = \text{const}$$

However in the general case $x \neq 1$ the second Kepler's law will be modified on the following manner: the radius - vector from the central body to the orbital body sweeps out equal areas in equal amount of time with increased ($x > 1$), or decresead ($x < 1$) continuation proportionally to the square root of the mass of the central body which is expressed in units of solar mass, i.e. [11, 12]:

$$\left(\frac{\pi\, r_i^2}{T_i\,.\sqrt{x\neq 1}}\right)^1 = \text{const}$$

The particular case $x = 1$ relates to the Solar system or to the systems of the stars which have masses near to the mass of Sun.

Results for the mean distances r_i and fixed places a_i of the Uranian satellites

The calculation results from the algorithm for the macro parameters of satellites in Uranian system are presented in Table 1. The Uranian moons are listed here by orbital period, from shortest to longest. The explanations to the 15 columns of the Table 1 are the following:

In column 1 the serial number of the corresponding Uranian satellite is given;

In column 2 the satellite name is presented;

In column 3: $\tilde{x}_i$ is the argument of the power function $y = 2^{x-2}$;

In column 4: $\tilde{a}_i$ is the interpolation ordering satellite number, as the 21-th place is the end of the system;

In column 5: $\tilde{d}_i$ is the value of the interpolating function;

In column 6 is presented the formation of approximating linear functional

$l_i(\tilde{f}) = \pi\,.\,\tilde{d}_i\,.\,x$ is the value of the interpolation linear functional;

In column 7: $\tilde{\Delta}_i = (y_i - y_{i-1})$ is the approximating difference;

In column 8: $-\tilde{k}_i = \pi\, x\, \tilde{\Delta}_i$ is subtrahend factor;

In column 9 the formation of the lower and upper interval boundaries of $l_i(\tilde{f})$ are shown. The lower boundary is

$\tilde{o}_i = l_i(\tilde{f}) - \tilde{k}_i$, as $-\tilde{k}_i$ is the subtrahend factor (column 8);

In column 10: $[\tilde{o}_i,\ l_i(\tilde{f})]$ is the interval of the approximating linear functional;

In column 11: d_i is the function $l_i(f)/(\pi.M_U)$ which we approximate;

In column 12: $l_i(f) = 10\, r_i$ is function which we approach, as $l_i(f)$ is expressed in AU;

In column 13: r_i is the distance (km) between the i-th satellite and the central body Uranus [1, 4];

In column 14: r_i is the distance (AU) between the i-th satellite and the central body Uranus. In the calculations we always express r_i in AU;

In column 15: a_i is the fixed place of the i-th satellite.

Results for macro parameters of Uranian satellites on the basis of Kepler's second law

These results of the algorithm calculations are presented in Table 2, as the explanations are the following:

In column 1 a serial number of the corresponding Uranian satellite is presented;

In column 2 are given the names of the Uranian satellites;

In column 3 are presented their orbital periods (in years);

In column 4 is shown the linear interpolation functional

$$l_i(\tilde{f}) = \left(\frac{\pi r_i^2}{T_i\,\sqrt{x}}\right)^2$$

In column 5 is calculated the interpolation function

$$\tilde{d}_i = l_i(\tilde{f})/(\pi\, M_U)$$

which is argument in interpolation functional;

In column 6 is given the correction $l_i(f) = \pi^2\, r_i$ concerning the functional $l_i(f) = 10\, r_i$ (column 12 in Table 1);

In column 7 is presented the value of

$$d_i = \frac{\pi^2 r_i}{\pi\, M_U} = \frac{\pi r_i}{4.373\times 10^{-5}} = 71840.674 r_i$$

Results for the accuracy

In Table 3 are shown the statistical data for the degree of approximation of $l_i(\tilde{f}) = l_i(f)$, respectivly $\tilde{d}_i = d_i$, for each Uranian satellite.

In column 1 is calculated the accuracy (%) of $l_i(\tilde{f})$ (column 4 in Table 2) versus $l_i(f) = 10\, r_i$ (column 12 in Table 1);

In column 2 is calculated the accuracy (%) of $l_i(\tilde{f})$ (column 4 in Table 2) versus $l_i(f) = \pi^2\, r_i$ (column 6 in Table 2);

In column 3 is calculated the accuracy (%) of $\tilde{d}_i$ (column 5 in Table 2) versus d_i (column 7 in Table 2).

TABLE 1 (left side)
Model for the macro parameters of the satellites in the Uranian system presented as algorithm for determination of their mean distances r_i and fixed places a_i as i = 0, 1, 2, 3, …, 21

№	Satellite	$\tilde{x}_i$	$\tilde{a}_i$	$\tilde{d}_i$	$l_i(\tilde{f})$	$\tilde{\Delta}_i$	$\tilde{o}_i = l_i(\tilde{f}) - \tilde{k}_i$	$\tilde{o}_i$	$[\tilde{o}_i,\ l_i(\tilde{f})]$
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
		x_0	0	1,250	0,00017	0,250	0,00003	0,00017 - 0,00003	[0,00014 ; 0,00017]
		x_1	1	1,50	0,0002	0,250	0,00003	0,0002 - 0,00003	[0,00017 ; 0,0002]
		x_2	2	2	0,0003	0,50	0,00006	0,0003 - 0,00006	[0,0002 ; 0,0003]
		x_3	3	3	0,0004	1	0,0001	0,0004 - 0,0001	[0,0003 ; 0,0004]
		x_4	4	5	0,0007	2	0,0003	0,0007 - 0,0003	[0,0004 ; 0,0007]
		x_5	5	9	0,0012	4	0,0005	0,0012 - 0,0005	[0,0007 ; 0,0012]
1	Cordelia	x_6	6	17	0,0023	8	0,0011	0,0023 - 0,0011	[0,0012 ; 0,0023]
2	Ophelia	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
3	Bianca	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
4	Cressida	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
5	Desdemona	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
6	Juliet	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
7	Portia	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
8	Rosalind	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
9	Cupid	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
10	Belinda	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
11	Perdita	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
12	Puck	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
13	Mab	x_7	7	33	0,0045	16	0,0022	0,0045 - 0,0022	[0,0023 ; 0,0045]
14	‡Miranda	x_8	8	65	0,0089	32	0,0044	0,0089 - 0,0044	[0,0045 ; 0,0089]
15	‡Ariel	x_8	8	65	0,0089	32	0,0044	0,0089 - 0,0044	[0,0045 ; 0,0089]
16	‡Umbriel	x_9	9	129	0,0177	64	0,0088	0,0177 - 0,0088	[0,0089 ; 0,0177]
17	‡Titania	x_{10}	10	257	0,0353	128	0,0176	0,0353 - 0,0176	[0,0177 ; 0,0353]
18	‡Oberon	x_{10}	10	257	0,0353	128	0,0176	0,0353 - 0,0176	[0,0177 ; 0,0353]
		x_{11}	11	513	0,0705	256	0,0352	0,0705 - 0,0352	[0,0353 ; 0,0705]
		x_{12}	12	1025	0,1408	512	0,0703	0,1408 - 0,0703	[0,0705 ; 0,1408]
19	♠Francisco	x_{13}	13	2049	0,282	1 024	0,141	0,282 - 0,141	[0,141 ; 0,282]
20	♠Caliban	x_{14}	14	4097	0,563	2 048	0,281	0,563 - 0,281	[0,282 ; 0,563]
21	♠Stephano	x_{14}	14	4097	0,563	2 048	0,281	0,563 - 0,281	[0,282 ; 0,563]
22	♠Trinculo	x_{14}	14	4097	0,563	2 048	0,281	0,563 - 0,281	[0,282 ; 0,563]
23	♠Sycorax	x_{14}	14	4097	0,563	2 048	0,281	0,563 - 0,281	[0,282 ; 0,563]
24	Margaret	x_{15}	15	8 193	1,126	4 096	0,563	1,126 - 0,563	[0,563 ; 1,126]
25	♠Prospero	x_{15}	15	8 193	1,126	4 096	0,563	1,126 - 0,563	[0,563 ; 1,126]
26	♠Setebos	x_{15}	15	8 193	1,126	4 096	0,563	1,126 - 0,563	[0,563 ; 1,126]
27	♠Ferdinand	x_{15}	15	8 193	1,126	4 096	0,563	1,126 - 0,563	[0,563 ; 1,126]
		x_{16}	16	16 385	2,251	8 192	1,125	2,251 - 1,125	[1,126 ; 2,251]
		x_{17}	17	32 769	4,502	16 384	2,251	4,502 - 2,251	[2,251 ; 4,502]
		x_{18}	18	65 537	9,005	32 768	4,502	9,005 - 4,502	[4,503 ; 9,005]
		x_{19}	19	131 073	18,009	65 536	9,004	18,009 - 9,004	[9,005 ; 18,009]
		x_{20}	20	262 145	36,019	131 072	18,010	36,019 - 18,010	[18,009 ; 36,019]
		x_{21}	21	524 289	72,037	262 144	36,018	72,037 - 36,018	[36,019 ; 72,037]

The symbols here are the following: ‡ Major moon, ♠ Retrograde moons.
Order refers to the position among other moons with respect to their average distance from Uranus.

TABLE 1
Continuation (right side)

d_i	$l_i(f) = 10\ r_i$	r_i, *km*	r_i, *AU*	a_i
11	**12**	**13**	**14**	**15**
[1,000 ; …	[0,00014 ; …	[2 094,37 ; …	[0,000014 ; 0,000017]	0
[1,375 ; 1,75]	[0,00017 ; 0,0002]	[2 543,16 ; 2 991,96]	[0,000017 ; 0,00002]	1
[1,75 ; 2,50]	[0,0002 ; 0,0003]	[2 991,96 ; 4 487,94]	[0,00002 ; 0,00003]	2
[2,50 ; 4,0]	[0,0003 ; 0,0004]	[4 487,94 ; 5 983,9]	[0,00003 ; 0,00004]	3
[4,0 ; 7,0]	[0,0004 ; 0,0007]	[5 983,9 ; 10 471,85]	[0,00004 ; 0,00007]	4
[7,0 ; 13,0]	[0,0007 ; 0,0012]	[10 471,85 ; 17 951,74]	[0,00007 ; 0,00012]	5
24,229	3,329E-03	49 800	3,329E-04	6
26,172	3,596E-03	53 800	3,596E-04	7.1
28,799	3,957E-03	59 200	3,957E-04	7.2
30,065	4,131E-03	61 800	4,131E-04	7.3
30,509	1,092E-03	62 700	4,192E-04	7.4
31,332	4,305E-03	64 400	4,305E-04	7.5
32,162	4,419E-03	66 100	4,419E-04	7.6
34,010	4,673E-03	69 900	4,673E-04	7.7
36,390	5,000E-03	74 800	5,000E-04	7.8
36,630	5,033E-03	75 300	5,033E-04	7.9
37,176	5,108E-03	76 420	5,108E-04	7.10
41,841	5,749E-03	86 000	5,749E-04	7.11
47,547	6,533E-03	97 734	6,533E-04	7.12
63,195	8,683E-03	129 900	8,683E-04	8.1
92,868	1,276E-02	190 900	1,276E-03	8.2
129,403	1,778E-02	266 000	1,778E-03	9
212,227	2,916E-02	436 300	2,916E-03	10.1
283,843	3,900E-02	583 500	3,900E-03	10.2
[385 ; 769]	[0,0353 ; 0,0705]	[528080,5 ; 1054665]	[0,00353 ; 0,00705]	11
[769 ; 1537]	[0,0705 ; 0,1408]	[1054665 ; 2107634]	[0,00705 ; 0,01408]	12
2 080,058	2,858E-01	4 276 000	2,858E-02	13
3 518,195	4,834E-01	7 231 000	4,834E-02	14.1
3 893,741	5,350E-01	8 004 000	5,350E-02	14.2
4 137,555	5,685E-01	8 504 000	5,685E-02	14.3
5 925,036	8,141E-01	12 179 000	8,141E-02	14.4
6 978,894	9,589E-01	14 345 000	9,589E-02	15,1
7 933,042	1,090	16 256 000	0,109	15,2
8 442,504	1,160	17 418 000	0,116	15,3
10 189,229	1,400 …	20 901 000	0,140	15,4
[12 289 ; 24 577]	[1,688 ; 3,377]	[… ; 25 252 120,5	[0,1688 ; 0,3377]	16
[24 577 ; 49 153]	[3,377 ; 6,754]	, … ,	[0,3377 ; 0,6754]	17
[49 153 ; 98 305]	[6,754 ; 13,507]	, … ,	[0,6754 ; 1,3507]	18
[98 305 ; 196 609]	[13,507 ; 27,014]	, … ,	[0,3507 ; 2,7014]	19
[196 609 ; 393 217]	[27,014 ; 31,180]	466 446 158,7]	[; 3,118] , …	20
				21

Table 2
Algorithm for determination of the macro parameters of the satellites in the Uranian system on the basis of Kepler's second law for every i = 0, 1, 2, 3, …, 21

Table 3
Statistical data for degree of approximation of $l_i(\tilde{f}) = l_i(f)$, respectivly $\tilde{d}_i = d_i$ for each Uranian satellite

№	Satellite	T_i, years	$l_i(\tilde{f})$	$\tilde{d}_i$	$l_i(f) = \pi^2 r_i$	d_i	Accuracy, %	Accuracy, %	Accuracy, %
1	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**
1	Cordelia	9,172E-04	3,295E-03	23,981	3,286E-03	23,915	98,979	99,727	99,725
2	Ophelia	1,029E-03	3,564E-03	25,939	3,549E-03	25,830	99,110	99,579	99,580
3	Bianca	1,191E-03	3,901E-03	28,392	3,905E-03	28,424	98,585	99,898	99,887
4	Cressida	1,270E-03	4,075E-03	29,658	4,077E-03	29,673	98,644	99,951	99,983
5	Desdemona	1,298E-03	4,137E-03	30,080	4,137E-03	30,112	98,664	100,000	99,894
6	Juliet	1,350E-03	4,253E-03	30,953	4,249E-03	30,923	98,792	99,906	99,903
7	Portia	1,405E-03	4,360E-03	31,732	4,361E-03	31,742	98,665	99,977	99,968
8	Rosalind	1,528E-03	4,609E-03	33,544	4,612E-03	33,567	98,630	99,935	99,931
9	Cupid	1,643E-03	5,225E-03	38,028	4,935E-03	35,916	95,694	94,450	94,446
10	Belinda	1,664E-03	5,227E-03	38,042	4,967E-03	36,153	96,288	95,026	95,034
11	Perdita	1,707E-03	5,271E-03	38,362	5,041E-03	36,691	96,908	95,636	95,644
12	Puck	2,086E-03	5,666E-03	41,237	5,674E-03	41,296	98,556	99,859	99,857
13	Mab	2,408E-03	6,440E-03	46,870	6,448E-03	46,927	98,576	99,876	99,879
14	‡Miranda	3,860E-03	8,510E-03	61,936	8,570E-03	62,371	98,008	99,300	99,303
15	‡Ariel	6,900E-03	1,257E-02	91,412	1,259E-02	91,656	98,511	99,841	99,734
16	‡Umbriel	1,133E-03	1,757E-02	127,875	1,755E-02	127,716	98,819	99,886	99,876
17	‡Titania	2,385E-03	2,869E-02	208,806	2,878E-02	209,459	98,388	99,687	99,688
18	‡Oberon	3,685E-03	3,845E-02	279,840	3,849E-02	280,131	98,590	99,896	99,896
19	♠Francisco	0,730	0,283	2 059,680	0,282	2 052,932	99,020	99,647	99,672
20	♠Caliban	1,587	0,489	3 558,952	0,477	3 472,313	98,855	97,546	97,566
21	♠Stephano	1,855	0,537	3 908,297	0,528	3 842,962	99,628	98,324	98,328
22	♠Trinculo	2,078	0,546	3 973,799	0,562	4 083,596	95,884	97,153	97,311
23	♠Sycorax	3,527	0,797	5 800,582	0,803	5 847,767	97,900	99,253	99,193
24	Margaret	4,503	0,941	6 848,617	0,946	6 887,880	98,133	99,471	99,430
25	♠Prospero	5,414	1,087	7 911,208	1,076	7 829,586	99,725	98,988	98,968
26	♠Setebos	5,985	1,141	8 304,221	1,145	8 332,403	98,362	99,651	99,662
27	♠Ferdinand	7,730 …	1,451 …	10560,408 …	1,382 …	10056,348 …	96,485	95,245	95,227
	$\tilde{a}_i$ = ai = 16	[... 10,467	[… 1,672	[.. 12168,850	[.. 1,666	[.. 12125,083			
	$\tilde{a}_i$ = ai = 17	, … ,	, … ,	, … ,	, … ,	, … ,			
	$\tilde{a}_i$ = ai = 18	, … ,	, … ,	, … ,	, … ,	, … ,			
	$\tilde{a}_i$ = ai = 19	, … ,	, … ,	, … ,	, … ,	, … ,			
	$\tilde{a}_i$ = ai = 20	832,466]	30,781]	224 024,745]	30,773]	223 969,245]			
	$\tilde{a}_i$ = ai = 21								
							98,607 *	98,804 *	98,799 *

The symbols here are the following: ‡ Major moon, ♠ Retrograde moons. The Uranian moons are listed here by distances r_i and orbital period, from shortest to longest.
* Arithmetical mean level of approximation for all moons (from 1 to 27)

Some satellites have the same quantum number i = 7, 8, 10, 14 and 15 because of the short distances between their orbits, i.e. the orbits have almost equal major semiaxis.

On the other hand some quantum numbers i remain not fulfilled (i = 11, 12 etc.), which means that there are yet not discovered satellites. The places where they are situated are predicted in the Tables 1 and 2. It should be noticed that the region x_{16} - x_{21} is yet not investigated.

Conclusion

New algorithm for the distances and fixed places of the satellites in Uranian system is proposed in the present paper. The Titius-Bode law in its classical form, represents only a part, or a fragment, of the here proposed generalized model.

This model is presented as an algorithm in Tables 1 and 2. It describes well the observed macrostructure and macro parameters (shown in Tables 1 and 2) of the 27 satellites in the Uranian system. The classical Titius-Bode law leads to some inaccuracy or deviation from the observed data [10, 13, 14]. In this relation the presented new improved algorithm for the Uranian-centric distances and fixed places of the satellites in Uranian system is more precise and possesses the corresponding physical meaning. It seems also possible to investigate the distances of Uranus rings from the planet, because of their sharply determined distances [14]. Our model has the advantage, that he gives also the intervals of the distances, hence, we can have information about the width of the rings. There are some difficulties with the rings of Saturn, which are very extended in the equatorial plane and distances from the planet are not well defined [14]. We will investigate in the near future not only the planetary satellites, but also the planetary rings. These studies are important for the physics of the whole Uranian system.

REFERENCES

1. Sheppard, S., D. Jewitt, J. Kleyna. Astronomical J., **129**, 2005, 518–525. doi:10.1086/426329.
2. Gazetteer of Planetary Nomenclature, "Planet and Satellite Names and Discoverers", USGS Astrogeology. July 21 2006.
3. Smith, B.A., L.A. Soderblom, A. Beebe, et al. Science, **233,** 1986, 97–102. doi:10.1126/science.233.4759.43. PMID 17812889.
4. http://en.wikipedia.org/wiki/Moons_of_Uranus
5. Showalter, M.R., J.J. Lissauer. Science, **311**, 2006, 973–977. doi:10.1126/ science.1122882. PMID 16373533.
6. Esposito, L. W. Planetary rings (pdf). Reports On Progress In Physics, **65**, 2002, 1741–1783. doi:10.1088/0034- 4885/65/12/201.
7. Brown R.H., D.P. Cruikshank, D. Morrison. Nature, **300**, 1982, 423 – 425. doi:10.1038/300423a0
8. Tyler, G.L., D.L. Sweetnam, J.D. Anderson et al. Science, **246**, 1989, 1466–1473. doi:10.1126/science.246.4936.1466. PMID 17756001.
9. Jacobson, R.A., J.K. Campbell, A.H. Taylor, S.P. Synnott. Astronomical J., **103**, 1992 (6), 2068–78. doi:10.1086/116211.
10. Velinov P.I.Y., D. Yuskolov. Compt. rend. Acad. bulg. Sci. **62**, 2009, No 7, 783 - 790.
11. Velinov P.I.Y., D. Yuskolov. Compt. rend. Acad. bulg. Sci. **62**, 2009, No 10, 1193 - 1202.
12. Velinov P.I.Y., D. Yuskolov. Compt. rend. Acad. bulg. Sci. **62**, 2009, No 11, 1353 - 1362.
13. Poveda A., P. Lara. Revista Mexicana de Astronomia y Astrofisica, **44**, 2008, No 1, 243 - 246.
14. Panov K. Compt. rend. Acad. bulg. Sci., **62**, 2009, No 2, 143 - 152.