New Algorithm for the Orbital Distances Law in Solar System and in Exo-planetary Systems

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New algorithm for the heliocentric distances and fixed places of the planets in Solar system and extra-solar planetary systems is proposed. The Titius-Bode law, which is not valid for Neptune, Pluto, Eris, etc., is generalized on this way. We propose linear quantization of space in the gravitational field of the Sun and the stars on the base of the exponential function $\tilde{d}_i = 2^{x-2} + C$ *, as C = 1 for every i = 1, 2, 3,…, 21 (i = 1 for Mercure, i = 2 for Venus, i = 3 for Earth, i = 4 for Mars, i = 5 for Ceres, i = 6 for Jupiter, i = 7 for Saturn, i = 8 for Uranus, i = 9 for Neptune, i = 10 for Pluto, i = 11 for Eris, etc.). This function* $\tilde{d}_i$ *multiplied with the mathematical constant* π *forms the interpolating linear functional* $l_i(\tilde{f})$ *, which contains information about the distance (in AU) of the i-th planet to the Sun, or the star, respectively. This functional is nonlinear because of the nonlinear dependence between the distance* r_i *from the Sun (the star) [in AU] and orbital period* T_i *(in years) according to the second Kepler law. After this law the radius vector* $/r/= r_i$ *from the Sun (the star) to the given planet describes equal surfaces for equal time intervals. The obtained algorithm can be applied for determination of the distances and fixed places of the planetary moons also.*

Introduction

The distances of the eight planets now known from the Sun range from 0.4 AU in the case of Mercury, to about ≈ 30 AU for Neptune and ≈ 40 AU for dwarf planet Pluto. In the 18th century Titius (1729-1796) and Bode (1747-1826), and later Wolf (1816-1893), showed that the average heliocentric distances of the planets approximately followed an empirical law. Originally applied to the planets from Mercury to Saturn, Uranus being added later, the law is given by [1, 2, 3, 4]:

$$r_i = 0.4 + 0.3 \times 2^n \qquad (1)$$

where r_i is the heliocentric distance expressed in AU, n takes the value of $-\infty$ for Mercury, 0 for Venus, and is increased by 1 for each successive planet (Table 1). The Titius-Bode law (TBl) is valid, with a moderate degree of error (less then 5%), as far as Uranus; but for Neptune the error increase is to 28.9%, and for Pluto and for Eris still more - to 95.4% and to 127.6%, respectively (Table 1). Thus, these errors increase with the growth of the heliocentric distances r_i . Note that the value $n = 3$ does not correspond to a planet as such, but to the approximate center of the asteroid belt. It is possible that the asteroids consist of partially accreted material that was not able to form a single body. This hypothesis would explain the existence of the asteroid belt at the heliocentric distance predicted by the Titius-Bode law.

The formula (1) given above, known as the Titius-Bode law, is far from exact, as can be seen by comparison with the observational data. Furthermore, despite persistent efforts of generations of scientists to discover a physical basis for this 'law', no fully satisfactory explanation has ever been proposed [1, 2, 3].

It is immediately seen also that for Mercury any effort to find a physical meaning of the parameter $n = -\infty$ would fail [5]. Because of that a lot of specialists give different approaches and solutions. In [6, 5] a new exponential TBl type relation for the Solar system is proposed. In the monograph [7] a logarithmic spiral formula is offered. The goal of the present paper is the presentation of a new improved algorithm for the heliocentric distances and fixed places of the planets in the Solar system. This algorithm must possess an increased accuracy.

New algorithm for the distances and fixed places of the planets

The first feature that can be seen in the TBl (1) is the presence of a geometric progression. The following exponential function corresponds to elements of this progression:

$$y = 2^x, \quad \text{as} \quad x = 0, 1, 2, \dots 19 \qquad (2)$$

In order to satisfy the needed properties of this function, i.e. to contain also the zero member, we must have

$$y = 2^{x-2}, \quad \text{as} \quad x = 0, 1, 2, \dots 21 \qquad (3)$$

To answer the question why we observe the specific Solar system structure with respective characteristic parameters, we have to follow and accomplish step by step the algorithm described in the columns in Tables 2 and 3, with the aid of function (3). This algorithm operates with the value of the mathematical constant π. This value converts the interpolation function denoted in Table 2 (column 10) with

$$\tilde{d}_i = 2^{x-2} + C, \quad \text{as} \quad C = 1 \qquad (4)$$

in an interpolating linear functional (column 11), which is denoted by $l_i(\tilde{f})$. The approximate relations are valid here

$$l_i(\tilde{f})_{11} \cong l_i(\tilde{f})_{18} = \left(\frac{\pi\, r_i^2}{T_i}\right)^2 \qquad (5)$$

as $i = 0, 1, 2, \dots 21$. Their values are near the values of the interpolation linear functional

$$l_i(f)_7 = 10\, r_i \cong l_i(f)_{19} = \pi^2 r_i = 9.870\, r_i \qquad (6)$$

or, in other words, the equality

$$l_i(\tilde{f}) = l_i(f)$$

is fulfilled. In (5, 6) r_i is the the absolute value of the radius-vector from the initial point of the 3D coordinate system *Oxyz* (the Solar centre) to the centre of the specified planet, which is a scalar value.

In Figure 1 the graphics of the interpolation linear functional $l_i(\tilde{f})$ (column 11 in Table 2) is presented, and the independent argument of the functional is the interpolation function (4) for each i = 0, 1 , …., 10. This function determines the values of the functional, as well as the exact fixed position of each planet (or of the set of bodies, as in the case of the asteroid belt).

The values of the independent variable of the interpolation function (4) are denoted by the symbol $\tilde{x}_i$, i = 0, 1, …., 10; they are given in the first line of values at the abscissa axis in Figure 1, and also in columns 8 and 9 in Table 2.

In the next line in Figure 1 below the axis $O\tilde{x}_i$ the values of the function $\tilde{d}_i$ are presented. The lower and, respectively, the upper boundaries of the intervals of variation of the values $\tilde{d}_i$ are given in the bottom line.

The object of our attention is the step-wise (or threshold) behavior of the curve a_i (column 16 in Table 2), which is determined by the discrete property of the positions of the planets. A fixed place of any planet can be only a whole natural number - under conditions of the continuing variations of the interpolation function $\tilde{d}_i$ (column 14 in Table 2). This leads to continuing variations of the values of the interpolation linear functional $l_i(\tilde{f})$. The graphic of the linear function $y = f(x) = | x |$ which is defined from the equation $\tilde{x}_i = a_i$ for each i = 0, 1, …., 10 is obtained on this way.

We see another feature of the graphics of a_i : the increase of values of $\tilde{x}_i$ along the abscissa leads to an increase also of the width of the intervals, in which the parameter a_i = constant (which determines the successive number of the specified body in orbit) does not change. These are the horizontal parts (plateaus) of a_i under conditions of the varying values of the interpolation function $\tilde{d}_i$ and the interpolation functional $l_i(\tilde{f})$ in the same intervals.

In other words, with the increase of the values of the parameter $\tilde{x}_i$, i = 1, 2, …10 the parameter a_i comprises bigger intervals of $l_i(\tilde{f})$ values and takes bigger space areas, where the force of the gravitational field **F** between the central body Sun ⊙ and the planets (incl. dwarf planets) acts.

A typical example for this are the two planets Uranus and Neptune, for which the parameter a_i obtains values, respectively 8.1 and 8.2, i.e. both planets occupy the 8-th position in the Solar system.

The hypothetical planet Vulcan is given at $a_i = 0$ in Table 2 (in brackets and with a questional sign). It is situated between the Sun and Mercury, i.e. Vulcan is not yet discovered intra-Mercurial planet or planetoid. But it is not out of consideration and it is object of searching [8]. It is possible that it had existed in the initial phase of the Solar system formation. Our algorithm gives its place at $a_i = 0$.

Observing a planet inside the orbit of Mercury is extremely difficult, since the telescope must be pointed very close to the Sun, where the sky is never black. Also, an error in pointing the telescope can result in damage for the optics, and injury to the observer. The huge amount of light present even quite away from the Sun can produce false reflections inside the optics, thus fooling the observer into seeing things that do not exist.

The best strategy for observations is to wait for the planet transit on the Sun disk. A small, round dark spot can be seen moving, as happens regularly with Mercury and Venus.

In 1915, when Einstein successfully explained the apparent anomaly in Mercury's orbit, most astronomers abandoned the search for Vulcan. A few, however, remained convinced that not all the alleged observations of Vulcan were bogus. Among these was H.C. Courten (Dowling College, New York) [8]. Studying photographic plates of the 1970 eclipse of the Sun, he and his associates detected several objects which appeared to be in orbits close to the Sun (Miami Herald, 15 June 1970). Even accounting for artifacts, H.C. Courten felt that at least seven of the objects were real. The appearance of some of these objects was confirmed by another observer in North Carolina, while a third observer in Virginia saw one of them.

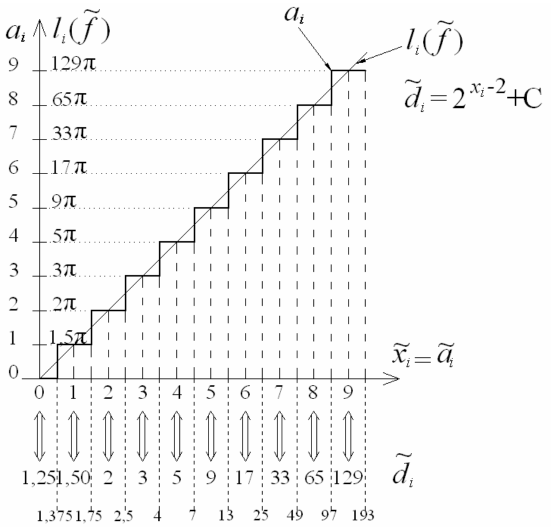

Figure 1

H.C. Courten believed that an intra-Mercurial planetoid between 130 and 800 km in diameter was orbiting the Sun at a distance of about 0.1 AU. Other images on his eclipse plates led him to postulate the existence of an asteroid belt between Mercury and the Sun [8].

None of these claims has ever been substantiated after more than thirty years of observation. It has been surmised, however, that some of these objects - and other alleged intra-Mercurial planets - may exist, being nothing more than previously unknown comets or small asteroids. Today, the search continues for these so-called Vulcanoid asteroids , which are thought to exist in the region (0.06 – 0.21 AU) where Vulcan was once sought. None have been found yet and searches have ruled out any such asteroids larger than about 60 km [3, 8].

TABLE 1.
Titius-Bode law for the planets in the Solar system (Ceres, Pluto and Eris are dwarf planets)

1	2	3 ?	4	5	6	7
Planet	Symbol	r_i	n	T-Br (1)	Error, %	$l_i = 10.r_i$
Mercury	☿	0.387	$-\infty$	0.4	3.4	3.87
Venus	♀	0.723	0	0.7	3.3	7.23
Earth	♁	1.000	1	1.0	0	10.00
Mars	♂	1.524	2	1.6	4.9	15.24
(Ceres)	⚳	2.760	3	2.8	1.4	27.60
Jupiter	♃	5.203	4	5.2	0.6	52.03
Saturn	♄	9.546	5	10.0	4.8	95.46
Uranus	♅	19.200	6	19.6	2.1	192.00
Neptune	♆	30.090	7	38.8	28.9	300.90
(Pluto)	♇	39.500	8	77.2	95.4	395.00
(Eris)		67.665	9	154	127.6	676.65

The planets (and dwarf planets Ceres, Pluto and Eris) are listed here by their average distances r_i from the Sun.

TABLE 2.
Model for the macro parameters of the planets in the Solar system presented as algorithm for determination of their heliocentric distances and fixed places

1	8	9	10	11	12	13	14	15	16
Planet	$\tilde{x}_i$	$\tilde{a}_i$	$\tilde{d}_i$	$l_i(\tilde{f})$	$\tilde{\Delta}_i$	$\tilde{o}_i/\pi, d_i(\tilde{f})$	$\tilde{d}_i$	d_i	a_i
Vulcan ?	x_0	0	1.25	3.927	0.25	[1, 1.25]	-	(1.000)	0
Mercury	x_1	1	1.50	4.712	0.25	[1.25, 1.5]	1.50	1.232	1
Venus	x_2	2	2	6.283	0.50	[1.5, 2]	2	2.301	2
Earth	x_3	3	3	9.425	1	[2, 3]	3	3.183	3
Mars	x_4	4	5	15.708	2	[3, 5]	5	4.851	4
(Ceres)	x_5	5	9	28.274	4	[5, 9]	9	8.795	5
Jupiter	x_6	6	17	53.407	8	[9, 17]	17	16.880	6
Saturn	x_7	7	33	103.672	16	[17, 33]	33	30.386	7
Uranus	x_8	8	65	204.203	32	[33, 65]	65	61.116	8.1
Neptune	x_9	9	129	405.265	64	[65, 129]	65	95.780	8.2
(Pluto)	x_{10}	10	257	807.389	128	[129, 257]	129	125.733	9
(Eris)	x_{11}	11	513	1611.636	256	[257, 513]	257	≈ 257	10
	x_{12}	12	1025	3220.129	512	[513, 1025]	513	-	11
	x_{13}	13	2049	6437.118	1024	[1025, 2049]	1025	-	12
	x_{14}	14	4097	12871.094	2048	[2049, 4097]	2049	-	13
	x_{15}	15	8193	25739.047	4096	[4097, 8193]	4097	-	14
	x_{16}	16	16385	51474.952	8192	[8193, 16385]	8193	[6366, 16385]	15
	x_{17}	17	32769	102946.763	16384	[16385, 32769]	16385	[16385, 32769]	16
	x_{18}	18	65537	205890.384	32768	[32769, 65537]	32769	[32769, 63632]	17
	x_{19}	19	131073	411777.626	65536	[65537, 131073]	65537	[63632, 159155]	18
	x_{20}	20	262145	823552.111	131072	[131073;262145]	131073	[159155, 5.E+5]	19
	x_{21}	21	524289	1647101.08	262144	[262145;524289]	262145	-	20
	-	-	-	-	-	-	524289	-	21

Order refers to the position among other planets with respect to their average distance r_i from the Sun.

TABLE 3.
Algorithm for determination of the macro parameters of planets in the Solar system on the basis of Kepler's second law and the corresponding statistical data for degree of approximation of $l_i(f)$

1		17	18	19	20	21	7
Planet		T_i, years	$l_i(\tilde{f})$	$l_i(f)=\pi^2 r_i$	Accuracy, %	Accuracy, %	$l_i = 10.r_i$
Mercury	☿	0.241	3.844	3.820	99.328	99.376	3.87
Venus	♀	0.615	7.130	7.136	98.617	99.916	7.23
Earth	♁	1.000	9.870	9.870	98.700	100.00	10.00
Mars	♂	1.881	15.047	15.041	98.734	99.960	15.24
(Ceres)	⚳	4.600	27.066	27.340	98.065	98.998	27.60
Jupiter	♃	11.862	51.404	51.351	98.797	99.897	52.03
Saturn	♄	29.458	94.445	94.215	98.937	99.756	95.46
Uranus	♅	84.018	190.002	189.496	98.959	99.734	192.00
Neptune	♆	164.780	297.974	296.976	99.028	99.665	300.90
Pluto	♇	248.400	389.389	389.849	98.579	99.882	395.00
					98.774*	99.818*	

The planets are listed here by orbital period, from shortest to longest.
* Arithmetical mean level of approximation for all planets.

Accuracy

Columns 20 and 21 in Table 3 demonstrate the results of the detailed statistic analysis of the accuracy of coincidence, which is evaluated as the approximation level in percent of the interpolation linear functional $l_i(\tilde{f})$ (column 18 in Table 3) related to:

(1) the classical linear functional $l_i(f) = 10\ r_i$ (column 7 in Table 1) and

(2) the corrected approximated linear functional presented in column 19 in Table 3.

It is seen that after this last correction the arithmetical mean level of approximation of the functionals is increased from 98.774% to 99.818% for all planets of the Solar system. This is shown in the additional rows in columns 20 and 21 in Table 3.

Conclusion

The studies in this work show that the Titius-Bode law in its classical form (1), represents only a part, or a fragment, of the here proposed generalized mathematical model. This model is presented as an algorithm in Tables 2 and 3 and it is illustrated by Figure 1. It describes well the observed macrostructure and macro parameters of the eight planets and of both dwarf planets (Ceres and Pluto) in the Solar system, which are shown in Tables 1, 2 and 3.

The Titius-Bode law leads to a considerable inaccuracy or deviation from the observed data, especially for the planet Neptune and for the dwarf planets Pluto, Eris, etc. (Table 1). The TBl error increases with the growth of the heliocentric distances r_i . In this relation the presented new improved algorithm for the heliocentric distances and fixed places of the planets in the Solar system [10] is much more precise and possesses the corresponding physical meaning. This method can be used at the investigation of the extra-solar planetary systems also. The developed here algorithm can be applied for determination of the distances and fixed places of the moons of Jupiter [11], Neptune [12], Uranus [13], Saturn [14] etc., and the moons of the exo-planetary systems.

REFERENCES

[1] Bakich M.E., The Cambridge Planetary Handbook, Cambridge University Press, 2000.
[2] Weissmann P.R., L.A. McFadden, T.V. Johnson. Encyclopedia of the Solar System, Academic Press, San Diego, 2007.
[3] Lewis J.S. Physics and Chemistry of the Solar System. Academic Press, San Diego, 2004.
[4] Encrenaz T., J.-P. Bibring, M. Blanc. The Solar System. Springer Verlag, Berlin, Heidelberg, 1990.
[5] Panov K. C.r. Acad. bulg. Sci., 62, 2009, 143-152.
[6] Poveda A., P. Lara. Revista Mexicana de Astronomia y Astrofisica, 44, 2008, No.1, 243-246.
[7] Gogoshev M. Cosmos, Ecology, Civilization. Stara Zagora, Thracian University, 1998, 51-54.
[8] Courten. H.C. Vulcan (planet). http://www.chemistry daily. com/chemistry/Vulcan_(planet)
[9] Brunier S., Solar System Voyage. Cambridge University Press, 2005.
[10] Velinov P.I.Y., D. Yuskolov. C.r. Acad. bulg. Sci., 62, 2009, 783-790.
[11] Velinov P.I.Y., D. Yuskolov. C.r. Acad. bulg. Sci., 62, 2009, 1193-1202.
[12] Velinov P.I.Y., D. Yuskolov. C.r. Acad. bulg. Sci., 62, 2009, 1353-1362.
[13] Velinov P.I.Y., D. Yuskolov. C.r. Acad. bulg. Sci., 63, 2010, No. 1.
[14] Velinov P.I.Y., D. Yuskolov. C.r. Acad. bulg. Sci., 63, 2010, No. 2.