The Orbital Distances Algorithm in Planetary Systems: The Moons of Saturn

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The Titius-Bode law, which is generalized by Velinov and Yuskolov (2009) for the planets in the Solar system, is applied also for Saturnian moons. New algorithm for the distances and fixed places of these moons is proposed in the present paper. We suggest linear quantization of space in the gravitational field of Saturn on the base of the exponential function $\tilde{d}_i = 2^{x-2} + C$, *as C = 1, for every i = 1, 2, 3,…, 21 (i = 5 for Epimetheus, Janus and all Cronian shepherd moons: Pan, Daphnis, Atlas, Prometheus, Pandora, and Aegaeon; i = 6 for Mimas, Alkyonides (Methone, Anthe, and Pallene), Enceladus, and the Trojan moons: Tethys, Telesto, and Calypso; i = 7 for Rhea, and the Trojan moons: Dione, Helene, and Polydeuces; i = 8 for Titan, i = 9 for Hyperion, i = 10 for Iapetus, i = 12 for Inuit, Gallic and part of Norse groups of satellites, i = 13 for the rest of Norse group, etc.). The function* $\tilde{d}_i$ *multiplied with the mathematical constant* π *forms the interpolating linear functional* $l_i(\tilde{f})$, *which contains information about the distance of the i-th satellite to Saturn. This functional is nonlinear because of the nonlinear dependence between the distance* r_i *from Saturn and orbital period* T_i *according to the second Kepler law. A lot of satellites have the same quantum number i because of almost equal distances from the central planet Saturn. Some quantum numbers i remain not fulfilled (for example i = 11) which means that there are yet not discovered moons. The places where they are situated are predicted in this paper.*

Introduction

In the study of Velinov and Yuskolov [1] a new generalized algorithm is proposed for the Titius-Bode law for the planets in the Solar system. As is well known the Titius-Bode law is an empirical algorithm, which we tried to improve with the corresponding linear quantization of space. This problem is not yet solved for the moons orbiting around the planets in the Solar system [2]. One attempt in that field was made by Panov [3]. We tried to apply our new algorithm not only for the planets, but for the planetary moons also, using the data from [4, 5]. Really the generalized Titius-Bode law for the planets in the Solar system [1], was applied successfully for Jovian [6], Neptunian [7] and Uranian [8] moons. The goal of the present work is the further enlargement of these applications, namely by means of quantitative consideration of the distances of the satellites in the Saturnian system.

In the paper of Panov [3] are considered 22 satellites of Saturn. The results for the first (n = 1-7) and foorth (n = 18-22) groups of moons are comparatively good (the errors are less than 5%). But the errors increase in the second (n = 8-14) and the third (n = 15-17) groups. For example, the errors for the biggest and most essential moons are the following: Mimas -13%, Enceladus 13%, Tethys 9%, Dione 1%, Rhea -14%, Titan -11%, Hyperion 26%, Iapetus -11%, etc. Data for all 22 investigated satellites are listed in Table 3 [3]. Panov [3] remarks for this system that “it is not possible to find a single exponential formula for all satellites of Saturn”.

That is why the problem with radial distances of Saturnian satellites remains yet open.

Saturnian moons

Saturn has 61 moons with confirmed orbits, 52 of which have names, and most of which are quite small. There are also hundreds of known moonlets embedded within Saturn's rings. With 7 major moons: Mimas, Enceladus, Tethys, Dione, Rhea, Titan and Iapetus, that are large enough to have sufficient gravitational attraction to become spherical in shape (and which would thus be considered dwarf planets if they were in direct orbit about the Sun) in addition to the planet’s broad and dense rings, the Saturnian system is the most diverse in the Solar system. Particularly notable are Titan, Rhea and Enceladus. The enormous **Titan**, at 5151 km diameter, is the second largest moon in the Solar System, with an Earth-like atmosphere and a landscape including hydrocarbon lakes at Titan's north and south poles and river networks. Titan may also possess a weak magnetic field. **Rhea**, at 1528 km diameter, is second largest Saturn’s moon and the ninth largest moon in the Solar System. It is the only moon in our system known so far to have a ring. **Enceladus** is the most geologically active of Saturn’s inner moons. The south pole of Enceladus is covered with fractures called "tiger stripes", some of which emit the particles that replenish the E-ring. Enceladus is the smallest known body in the Solar System that is geologically active today, and it may have liquid water underneath the south-polar surface [4].

23 of Saturn's moons are *regular satellites*, with prograde orbits that are not greatly inclined with respect to Saturn's equatorial plane. In addition to the 7 major satellites, an additional 4 moons are small Trojans that share an orbit with a larger moon. The regular satellites are traditionally named after Titans or other figures associated with the mythological Saturn [4, 5].

The remaining 38, all small, are *irregular satellites*, whose orbits are much farther from Saturn, have high inclinations, and are mixed between prograde and retrograde. These moons were likely captured minor planets, or debris from the breakup of such bodies after they were captured, creating coalitional families. The irregular satellites have been classified by their orbital characteristics into Inuit, Norse, and Gallic groups, and their names are chosen from the corresponding mythologies [4, 5].

***Orbital groups* [4].** Although the borders may be somewhat nebulous, Saturn's moons can be divided into **ten groups**

according to their orbital characteristics. Many of Saturn's moons, such as Pan and Daphnis, orbit within Saturn's ring system and have orbital periods only slightly longer than the planet's rotation period. The innermost moons and most regular satellites all have inclinations ranging from less than a degree to ~1.5 degrees (the exception is Iapetus, which has an inclination of 7.57 degrees). But several irregular satellites in the outermost regions of Saturn's moon system, in particular the Norse group, have orbital radii of millions of km and orbital periods lasting several years. The moons of the Norse group also lie almost perpendicular to Saturn's equator, while the moons of the Gallic and Inuit groups have inclinations ranging from 30 to 50 degrees.

Ring shepherds. Shepherd satellites are moons that orbit within, or just beyond, a planet's ring system. They have the effect of sculpting the rings: giving them sharp edges, and creating gaps between them. Cronian shepherd moons are the moonlets, Pan, Daphnis, Atlas, Prometheus, Pandora, Aegaeon, in addition to the unconfirmed moons S/2004 S4, S/2004 S6 and S/2004 S3.

Inner large moons and the Alkyonides. The first from this group is Mimas, which is noticeably ovoid-shaped, having been made shorter at the poles and longer at the equator by the effects of Saturn's gravity. Mimas is the least massive of the inner moons, although its mass is sufficient to alter the orbit of Methone.

Alkyonides: These three moons orbit between Mimas and Enceladus: Methone, Anthe, and Pallene. They are some of the smallest moons in the Saturn system. Alkyonides possess very faint ring arcs.

The following moons are Enceladus and Tethys, which is the third largest of Saturn's inner moons. The moon's most prominent features are a very large crater named Odysseus and a vast canyon system named Ithaca Chasma.

Dione is the second-largest inner moon of Saturn. It does have a higher density than Rhea, the largest inner moon. Dione is covered with areas of bright fractures called "wispy terrain". This moon may also be geologically active.

Rhea, is the largest of Saturn's inner moons. Rhea most likely has a very faint ring system. The Cassini Orbiter detected an absence of electrons near Rhea's Hill sphere during a 2005 flyby, which is most likely caused by the presence of dust-sized ring particles. There is a very large crater on Rhea's leading hemisphere that is much brighter than the surrounding area.

Trojan moons. Trojan moons are a unique feature not yet found outside the moons of the Saturn system. Trojan bodies orbit at the Lagrange points of a much larger object, such as a large moon or planet. Tethys has two trojan moons, Telesto and Calypso, and Dione also has two, Helene and Polydeuces.

Outer large moons. These moons all orbit beyond the E Ring. They are:

Titan, which has an asynchronous orbit (when a moon does not always show the same side to its planet), and this may be explained by the subsurface ocean.

Hyperion is Titan's nearest neighbor in the Saturn system. Smaller than Mimas, Hyperion is one of the largest known non-spherical objects discovered so far. Hyperion is also the only moon known to have a chaotic rotation, which means the moon has no well-defined poles or equator. It is widely suspected that Hyperion is a fragment of a much larger moon that was destroyed earlier in the Saturn-Titan system's formation.

Iapetus is the third-largest of Saturn's moons. Unlike the other large moons, Iapetus orbits outside of Saturn's magnetic field, so it is constantly exposed to weak solar wind and cosmic radiation. Iapetus may be a captured Kuiper Belt Object.

Discoveries of the moons.

Before the advent of telescopic photography, 8 moons of Saturn were discovered by direct observation using an optical telescope. Saturn's largest moon, Titan, was discovered in 1655 by Huygens. Tethys, Dione, Rhea and Iapetus (the "Sidera Lodoicea") were discovered 1671-1684 by Cassini. Herschel discovered Mimas and Enceladus in 1789. Hyperion was discovered 1848 by the British team: W.C. Bond, G.P. Bond and W. Lassell [4, 9].

The use of long-exposure photographic plates made it possible to discover additional moons. The first to be discovered in this manner, Phoebe, was found in 1899 by W.H. Pickering. In 1966, the satellites Janus and Epimetheus were observed, but not confirmed, and it was not realized that there were two distinct moons sharing an orbit [5, 9].

Observations by spacecrafts.

The study of the outer planets has since been revolutionized by the use of unmanned space probes. The arrival of the Voyager space probes at Saturn in 1980 resulted in the discovery of 7 additional moons, bringing the total from 10 to 17. Epimetheus was confirmed as distinct from Janus. In 1990, Pan was discovered in archival Voyager images [4].

The Cassini mission, which arrived at Saturn in the summer of 2004, discovered 3 small moons in the inner Saturnian system (Methone, Pallene and Polydeuces) as well as 3 suspected but unconfirmed moons in the F Ring. On November 16, 2004 Cassini scientists announced that the structure of Saturn's rings indicates the presence of several more moons orbiting within the rings, but only one, Daphnis, has been visually confirmed so far (its confirmation was announced on May 6, 2005) [10]. On July 18, 2007 Anthe was announced. On March 6, 2008 it was announced that *Cassini* observations of a depletion of energetic electrons in Saturn's magnetosphere near Rhea might be the signature of a tenuous ring system around Saturn's second largest moon [11]. On March 3, 2009, Aegaeon, a moonlet within the G Ring, was announced.

TABLE 1 (left side)
Model for the macro parameters of the satellites in the Saturnian system presented as algorithm for determination of their mean distances r_i and fixed places a_i as i = 0, 1, 2, 3, …, 21

№	Satellite	$\tilde{x}_i$	$\tilde{a}_i$	$\tilde{d}_i$	$l_i(\tilde{f})$	$\tilde{\Delta}_i$	$-\tilde{k}_i$	$\tilde{o}_i = l_i(\tilde{f}) - \tilde{k}_i$	$[\tilde{o}_i,\ l_i(\tilde{f})]$
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
		x_0	0	1,250	1,122.E-03	0,250	0,0002245	0,001122 - 0,0002245	[0,8975 ; 1,122]E-03
		x_1	1	1,50	1,347.E-03	0,250	0,0002245	0,001347- 0,0002245	[1,122 ; 1,347]E-03
		x_2	2	2	1,796.E-03	0,50	0,0004489	0,001796 - 0,0004489	[1,347 ; 1,796]E-03
		x_3	3	3	2,694.E-03	1	0,0008979	0,002694 - 0,0008979	[1,796 ; 2,694]E-03
		x_4	4	5	4,489.E-03	2	0,001795	0,004489 - 0,001795	[2,694 ; 4,489]E-03
1	Pan	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
2	Daphnis	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
3	Atlas	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
4	Prometheus	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
5	Pandora	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
6a	Epimetheus	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
6b	Janus	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
8	Aegaeon	x_5	5	9	8,081.E-03	4	0,003592	0,008081 - 0,003592	[4,489 ; 8,081]E-03
9	**Mimas**	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
10	Methone	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
11	Anthe	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
12	Pallene	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
13	**Enceladus**	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
14	**Tethys**	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
14a	Telesto	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
14b	Calypso	x_6	6	17	1,526.E-02	8	0,007183	0,01526 - 0,007183	[0,8081 ; 1,526]E-02
17	**Dione**	x_7	7	33	2,963.E-02	16	0,01437	0,02963 - 0,01437	[1,526 ; 2,963]E-02
17a	Helene	x_7	7	33	2,963.E-02	16	0,01437	0,02963 - 0,01437	[1,526 ; 2,963]E-02
17b	Polydeuces	x_7	7	33	2,963.E-02	16	0,01437	0,02963 - 0,01437	[1,526 ; 2,963]E-02
20	**Rhea**	x_7	7	33	2,963.E-02	16	0,01437	0,02963 - 0,01437	[1,526 ; 2,963]E-02
21	**Titan**	x_8	8	65	5,836.E-02	32	0,02873	0,05836 - 0,02873	[2,963 ; 5,863]E-02
22	Hyperion	x_9	9	129	0,116	64	0,057	0,116 - 0,057	[0,059 ; 0,116]
23	**Iapetus**	x_{10}	10	257	0,231	128	0,115	0,231 - 0,115	[0,116 ; 0,231]
	-	x_{11}	11	513	0,461	256	0,230	0,461 - 0,230	[0,231 ; 0,461]
24	Kiviuq	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
25	Ijiraq	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
26	♠ Phoebe	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
27	Paaliaq	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
28	♠ Skathi	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
29	Albiorix	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
30	♠S/2007 S2	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
31	Bebhionn	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
32	Erriapus	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
33	♠Skoll	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
34	Siarnaq	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
35	Tarqeq	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
36	♠S/2004S13	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
37	♠ Greip	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
38	♠Hyrrokkin	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
39	♠Jarnsaxa	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
40	Tarvos	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
41	♠Mundilfari	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
42	♠S/2006 S 1	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
43	♠S/2004S17	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
44	♠Bergelmir	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
45	♠Narvi	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
46	♠Suttungr	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
47	♠Hati	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
48	♠S/2004S12	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
49	♠Farbauti	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
50	♠Thrymr	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]

TABLE 1 (right side) **Model for the macro parameters of the satellites in the Saturnian system presented as algorithm for determination of their mean distances r_i and fixed places a_i as $i = 0, 1, 2, 3, \ldots, 21$**				
d_i	$l_i(f) = 10\ r_i$	r_i, *km*	r_i, *AU*	a_i
11	**12**	**13**	**14**	**15**
[1,000 ; ...	[0,0008975 ; …	[13426,409 ; …	[0,8975 ; 1,1122]E-04	0
[1,375 ; 1,75]	[0,001122 ;0,00135]	[16784,9 ;20150,8]	[1,1122 ; 1,347]E-04	1
[1,75 ; 2,5]	[0,001347 ; 0,001796]	[20150,8 ; 26867,8]	[1,347 ; 1,796]E-04	2
[2,5 ; 4,0]	[0,001796 ; 0,002694]	[26867,8 ; 40301,7]	[1,796 ; 2,694]E-04	3
[4,0 ; 7,0]	[0,002694 ; 0,004489]	[40301,7 ; 67154,5]	[2,694 ; 4,489]E-04	4
9,947	8,931.E-03	133 600	8,931.E-04	5.1
10,166	9,128.E-03	136 505	9,128.E-04	5.2
10,252	9,205.E-03	137 700	9,205.E-04	5.3
10,378	9,318.E-03	139 400	9,318.E-04	5.4
10,549	9,472.E-03	141 700	9,472.E-04	5.5
11,271	1,012.E-02	151 422	1,012.E-03	5.6
11,282	1,013.E-02	151 472	1,013.E-03	5.7
12,474	1,120.E-02	167 500	1,120.E-03	5.8
13,799	1,239.E-02	185 404	1,239.E-03	6.1
14,478	1,300.E-02	194 440	1,300.E-03	6.2
14,723	1,322.E-02	197 700	1,322.E-03	6.3
15,804	1,419.E-02	212 280	1,419.E-03	6.4
17,719	1,591.E-02	237 950	1,591.E-03	6.5
21,929	1,969.E-02	294 619	1,969.E-03	6.6
21,929	1,969.E-02	294 619	1,969.E-03	6.7
21,929	1,969.E-02	294 619	1,969.E-03	6.8
28,099	2,523.E-02	377 400	2,523.E-03	7.1
28,099	2,523.E-02	377 400	2,523.E-03	7.2
28,099	2,523.E-02	377 400	2,523.E-03	7.3
39,236	3,523.E-02	527 100	3,523.E-03	7.4
90,968	8,168.E-02	1 221 930	8,168.E-03	8
110,258	9,900.E-02	1 481 010	9,900.E-03	9
265,063	0,238	3 560 820	2,380.E-02	10
[385 ; 769]	[0,231 ; 0,461]	[3455710,8 ;6896461,8]	[0,0231 ; 0,0461]	11
840,851	0,755	11 294 800	7,550.E-02	12.1
845,306	0,759	11 355 316	7,591.E-02	12.2
957,790	0,860	12 869 700	8,603.E-02	12.3
1124,847	1,010	15 103 400	0,1010	12.4
1167,168	1,048	15 672 500	0,1048	12.5
1210,603	1,087	16 266 700	0,1087	12.6
1232,877	1,107	16 560 000	0,1107	12.7
1277,425	1,147	17 153 520	0,1147	12.8
1282,983	1,152	17 236 900	0,1152	12.9
1300,813	1,168	17 473 800	0,1168	12.10
1323,087	1,188	17 776 600	0,1188	12.11
1333,111	1,197	17 910 600	0,1197	12.12
1344,248	1,207	18 056 300	0,1207	12.13
1345,361	1,208	18 065 700	0,1208	12.14
1352,044	1,214	18 168 300	0,1214	12.15
1381,000	1,240	18 556 900	0,1240	12.16
1382,114	1,241	18 562 800	0,1241	12.17
1394,365	1,252	18 725 800	0,1252	12.18
1408,843	1,265	18 930 200	0,1265	12.19
1422,207	1,277	19 099 200	0,1277	12.20
1422,207	1,277	19 104 000	0,1277	12.21
1443,368	1,296	19 395 200	0,1296	12.22
1457,846	1,309	19 579 000	0,1309	12.23
1466,756	1,317	19 709 300	0,1317	12.24
1482,348	1,331	19 905 900	0,1331	12.25
1487,916	1,336	19 984 800	0,1336	12.26
1509,077	1,355	20 278 100	0,1355	12.27

TABLE 1 (left side) **Continuation**									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
51	♠Aegir	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
52	♠S/2007 S3	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
53	♠Bestla	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
54	♠S/2004 S 7	x_{12}	12	1 025	0,920	512	0,459	0,920 - 0,459	[0,461 ; 0,920]
55	♠S/2006 S 3	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
56	♠Fenrir	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
57	♠Surtur	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
58	♠Kari	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
59	♠Ymir	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
60	♠Loge	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
61	♠Fornjot	x_{13}	13	2 049	1,840	1 024	0,920	1,840 - 0,920	[0,920 ; 1,840]
		x_{14}	14	4 097	3,679	2 048	1,839	3,679 - 1,839	[1,840 ; 3,679]
		x_{15}	15	8 193	7,356	4 096	3,677	7,356 - 3,677	[3,679 ; 7,356]
		x_{16}	16	16 385	14,712	8 192	7,356	14,712 - 7,356	[7,356 ; 14,712]
		x_{17}	17	32 769	29,423	16 384	14,711	29,423 - 14,711	[14,712 ; 29,423]
		x_{18}	18	65 537	58,846	32 768	29,423	58,846 -29,423	[29,423 ; 58,846]
		x_{19}	19	131 073	117,690	65 536	58,844	117,690 - 58,844	[58,846 ; 117,690]
		x_{20}	20	262 145	235,380	131 072	117,690	235,380 - 117,690	[117,690 ; 235,380]
		x_{21}	21	524 289	470,759	262 144	235,379	470,759 - 235,379	[235,380 ; 470,759]

Order refers to the position among other moons with respect to their average distance from Saturn. The moons are listed here by orbital period, from shortest to longest. Moons massive enough for their surfaces to have collapsed into a spheroid are in bold (Major moons). The irregular captured moons (beyond Iapetus) are shaded with ♠ when retrograde.

TABLE 1 (right side) **Continuation**				
11	**12**	**13**	**14**	**15**
1524,669	1,369	20 482 900	0,1369	12.28
1528,010	1,372	20 518 500	0,1372	12.29
1531,351	1,375	20 570 000	0,1375	12.30
1531,351	1,375	20 576 700	0,1375	12.31
1569,217	1,409	21 076 3000	0,1409	13.1
1632,699	1,466	21 930 664	0,1466	13.2
1659,428	1,490	22 288 916	0,1490	13.3
1661,655	1,492	22 321 200	0,1492	13.4
1669,451	1,499	22 429 673	0,1499	13.5
1710,658	1,536	22 984 322	0,1536	13.6
1824,257	1,638	24 504 879...	0,1638	13.7
[3073 ; 6145]	[2,759 ; 5,518]	[...41289012,12	[0,276 ; 0,552]	14
[6145 ; 12289]	[5,518 ;11,034]	, ... ,	[0,552 ; 1,103]	15
[12289 ; 24577]	[11,034 ; 22,068]	229333534,71]	[1,103 ; 1,533]	16
				17
				18
				19
				20
				21

TABLE 2
Algorithm for determination of the macro parameters of the satellites in the Saturnian system on the basis of Kepler's second law for every $i = 0, 1, 2, 3, \ldots, 21$

TABLE 3 Statistical data for degree of approximation of $l_i(\tilde{f}) = l_i(f)$, respectivly $\tilde{d}_i = d_i$ for each Saturnian satellite

№	Satellite	T_i, years	$l_i(\tilde{f})$	$\tilde{d}_i$	$l_i(f) = \pi^2 r_i$	d_i	Accuracy, %	Accuracy, %	Accuracy, %
1	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**
1	Pan	1,574.E-03	8,863.E-03	9,871	8,815.E-03	9,817	99,239	99,458	99,453
2	Daphnis	1,627.E-03	9,052.E-03	10,081	9,009.E-03	10,038	99,167	99,525	99,524
3	Atlas	1,648E-03	9,124.E-03	10,161	9,085.E-03	10,118	99,120	99,572	99,577
4	Prometheus	1,678.E-03	9,241.E-03	10,292	9,196.E-03	10,242	99,174	99,513	99,514
5	Pandora	1,722.E-03	9,369.E-03	10,434	9,348.E-03	10,411	98,913	99,776	99,780
6a	Epimetheus	1,901.E-03	1,002.E-02	11,159	9,988.E-03	11,124	99,012	99,681	99,686
6b	Janus	1,902.E-03	1,005.E-02	11,193	9,998.E-03	11,135	99,210	99,483	99,482
8	Aegaeon	2,213.E-03	1,109.E-02	12,351	1,105.E-02	12,311	99,018	99,639	99,676
9	**Mimas**	2,580.E-03	1,222.E-02	13,610	1,223.E-02	13,619	98,628	99,918	99,934
10	Methone	2,764.E-03	1,290.E-02	14,367	1,283.E-02	14,289	99,231	94,457	94,457
11	Anthe	2,838.E-03	1,309.E-02	14,578	1,305.E-02	14,531	99,017	99,694	99,678
12	Pallene	3,159.E-03	1,402.E-02	15,614	1,400.E-02	15,597	98,802	99,857	99,891
13	**Enceladus**	3,752.E-03	1,571.E-02	17,496	1,570.E-02	17,488	98,743	99,936	99,954
14	**Tethys**	5,169.E-03	1,942.E-02	21,628	1,943.E-02	21,643	98,629	99,949	99,931
14a	Telesto	5,169.E-03	1,942.E-02	21,628	1,943.E-02	21,643	98,629	99,949	99,931
14b	Calypso	5,169.E-03	1,942.E-02	21,628	1,943.E-02	21,643	98,629	99,949	99,931
17	**Dione**	7,493.E-03	2,491.E-02	27,743	2,490.E-02	27,732	98,732	99,960	99,960
17a	Helene	7,493.E-03	2,491.E-02	27,743	2,490.E-02	27,732	98,732	99,960	99,960
17b	Polydeuces	7,493.E-03	2,491.E-02	27,743	2,490.E-02	27,732	98,732	99,960	99,960

TABLE 2 Continuation (page 2 of 3)							**TABLE 3** Continuation(page 2 of 3)		
1	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**
20	**Rhea**	1,237.E-02	3,475.E-02	38,701	3,477.E-02	38,724	98,638	99,942	99,941
21	**Titan**	4,366.E-02	8,059.E-02	89,754	8,061.E-02	89,782	98,666	99,975	99,969
22	Hyperion	5,825.E-02	9,771.E-02	108,821	9,771.E-02	108,820	98,697	100,000	99,999
23	**Iapetus**	0,217	0,235	261,722	0,235	261,607	98,739	100,000	97,956
24	Kiviuq	1,227	0,745	829,714	0,745	829,887	98,675	100,000	99,979
25	Ijiraq	1,237	0,749	834,169	0,749	834,393	98,682	100,000	99,973
26	♠ Phoebe	1,492	0,849	945,540	0,849	945,631	98,721	100,000	99,990
27	Paaliaq	1,897	0,998	1111,482	0,996	1110,179	98,812	99,800	99,883
28	♠ Skathi	2,006	1,035	1152,690	1,034	1151,948	98,760	99,903	99,936
29	Albiorix	2,121	1,071	1192,883	1,073	1194,817	98,528	99,814	99,830
30	♠S/2007 S2	2,171	1,100	1225,081	1,093	1216,801	99,368	99,364	99,324
31	Bebhionn	2,296	1,133	1261, 833	1,132	1260,768	98,779	99,912	99,916
32	Erriapus	2,313	1,136	1265,174	1,137	1266,264	98,611	99,912	99,914
33	♠Skoll	2,361	1,152	1282,994	1,153	1283,851	98,630	99,913	99,933
34	Siarnaq	2,423	1,171	1304,153	1,172	1305,835	98,569	99,915	99,871
35	Tarqeq	2,450	1,180	1314,178	1,181	1315,727	98,580	99,915	99,882
36	♠S/2004S13	2,480	1,191	1326,428	1,191	1326,719	98,674	100,000	99,978
37	♠ Greip	2,482	1,193	1328,656	1,192	1327,818	98,758	99,916	99,937
38	♠Hyrrokkin	2,503	1,197	1333,111	1,198	1334,414	98,600	99,917	99,902
39	♠Jarnsaxa	2,584	1,222	1360,953	1,224	1362,992	98,548	99,837	99,850
40	Tarvos	2,585	1,225	1364,294	1,225	1364,092	98,711	100,000	99,985
41	♠Mundilfari	2,619	1,236	1376,545	1,236	1376,183	98,722	100,000	99,974
42	♠S/2006 S 1	2,662	1,247	1388,796	1,248	1390,472	98,577	99,920	99,879
43	♠S/2004S17	2,698	1,261	1404,388	1,260	1403,662	98,747	99,921	99,948
44	♠Bergelmir	2,699	1,260	1403,274	1,260	1403,662	98,669	100,000	99,972
45	♠Narvi	2,761	1,277	1422,207	1,279	1424,547	98,538	99,844	99,836
46	♠Suttungr	2,800	1,293	1440,027	1,292	1438,836	98,778	99,923	99,917

TABLE 2 Continuation (page 3 of 3)								**TABLE 3** Continuation (page 3 of 3)		
1	**2**		**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**
47	♠Hati		2,828	1,298	1445,595	1,300	1447,630	98,557	99,846	99,859
48	♠S/2004S12		2,871	1,314	1463,415	1,314	1463,019	98,723	100,000	99,980
49	♠Farbauti		2,888	1,318	1467,869	1,319	1468,514	98,653	99,924	99,956
50	♠Thrymr		2,952	1,335	1486,803	1,337	1489,399	98,524	99,850	99,826
51	♠Aegir		2,997	1,350	1503,505	1,351	1504,788	98,612	99,926	99,915
52	♠S/2007 S3		≈ 3,012	≈ 1,348	≈ 1501,281	1,354	1508,085	98,251	99,557	99,549
53	♠Bestla		3,016	1,356	1510,190	1,357	1511,383	98,618	99,926	99,921
54	♠S/2004 S 7		3,017	1,355	1509,077	1,357	1511,383	98,545	99,853	99,847
55	♠S/2006 S 3		3,128	1,390	1548,057	1,391	1548,755	98,652	99,928	99,955
56	♠Fenrir		3,320	1,446	1610,424	1,447	1611,409	98,636	99,931	99,939
57	♠Surtur		3,401	1,471	1638,267	1,471	1637,789	98,725	100,000	99,971
58	♠Kari		3,409	1,472	1639,381	1,473	1639,988	98,660	99,932	99,963
59	♠Ymir		3,434	1,478	1646,063	1,479	1647,682	98,599	99,932	99,902
60	♠Loge		3,562	1,514	1686,157	1,516	1688,352	98,568	99,809	99,870
61	♠Fornjot		3,921...	1,616 ...	1799,755...	1,617 ...	1800,469 ...	98,657	99,938	99,960
		14	[...8,575	[... 2,724	[...3033,745	[...2,724	[... 3033,757			
		15	, ... ,	, ... ,	, ... ,	, ... ,	, ... ,			
		16	112,246]	15,130]	16850,429]	15,130]	16850,544]			
		17	-							
		18	-							
		19	-							
		20	-							
		21	-							
								98,741 *	99,856 *	99,857 *

Ground-based observations. Study of Saturn's moons has also been aided by advances in telescopy. For the entire 20th century, Phoebe stood alone among Saturn's known moons in its highly irregular orbit. Beginning in 2000, three dozen additional irregular moons have been found using ground-based telescopes. A survey starting in late 2000 found 13 new moons orbiting Saturn at a great distance in orbits that suggest they are fragments of larger bodies captured by Saturn's gravitational pull [12]. On May 3, 2005, astronomers using the Mauna Kea Observatory announced the discovery of 12 more small outer moons [13, 14]. On June 30, 2006 astronomers using the Subaru 8.2 m telescope announced the discovery of 8 more small outer moons [15]. On April 13, 2007 Tarqeq and on May 1, 2007 S/2007 S 2 and S/2007 S 3 were announced [16, 17].

New algorithm for the distances and fixed places of the Saturnian moons

We suggest linear quantization of space in the gravitational field of Saturn on the basis of the exponential function [1]

$$\tilde{d}_i = 2^{x-2} + \mathrm{C}\ , \text{ as } \mathrm{C} = 1$$

for every i = 1, 2, 3,…, 21 (i = 5 for Epimetheus, Janus and all Cronian shepherd moons: Pan, Daphnis, Atlas, Prometheus, Pandora, and Aegaeon; i = 6 for Mimas, Alkyonides (Methone, Anthe, and Pallene), Enceladus, and the Trojan moons: Tethys, Telesto, and Calypso; i = 7 for Rhea, and the Trojan moons: Dione, Helene, and Polydeuces; i = 8 for Titan, i = 9 for Hyperion, i = 10 for Iapetus, i = 12 for Inuit, Gallic and part of Norse groups of satellites, i = 13 for the rest of Norse group, etc.). The function $\tilde{d}_i$ multiplied with the mathematical constant π forms the interpolating linear functional $l_i(\tilde{f})$, which contains information about the distance of the i-th satellite to Saturn. This functional is nonlinear because of the nonlinear dependence between the distance r_i from Saturn and orbital period T_i according to the second Kepler's law. After this law the radius vector $|\mathbf{r}| = r_i$ from Saturn to the given satellite describes equal surfaces for equal time intervals.

In our calculation we use several magnitudes and constants related to the sixth planet in our Solar system. The following values are very often used in the calculations in Tables 1 and 2:

1) The mass of Saturn is 1/3498,5 from the mass of the Sun M_S [4]. Otherwise we can write:

$$M_{SAT} = 2.858 \times 10^{-4}\ M_S$$

2) The square root from the Saturnian mass in units $M_S = 1$ is

$$(M_{SAT})^{1/2} = 1.691 \times 10^{-2}$$

3) The product of π with the Saturn's mass is

$$\pi\ M_{SAT} = 8.979 \times 10^{-4}\ M_S$$

For the Saturnian satellite system the general case of the second Kepler's law is valid which is characterized by the product [6, 7, 8]

$$T_i \times \sqrt{x \neq 1} \qquad (1)$$

where T_i is the orbital period of the i - th satellite (in years), $x = M_{SAT} \times (M_S = 1)$ is the mass of the central body, in our case - the planet Saturn, which mass is expressed in units of Solar mass (M_S = 1). The product (1) is divisor in the right side of the equation

$$l_i(\tilde{f}) = \left(\frac{\pi\ r_i^2}{T_i . \sqrt{x \neq 1}} \right)^2$$

in the case $x < 1$. r_i is the module (the magnitude) of the radius-vector (i.e. the distance) from i-th satellite until the planet Saturn, which is expressed in AU (1 AU=149 597 870 km [4]).

The square root from the mass of the central body when $x \neq 1$ changes the period of the tour of the satellite around Saturn. As it was noticed for the satellite system of Saturn $x < 1$ and the period decreases.

The mathematical model of the second Kepler's law for the Solar system (x = 1) is [1]

$$\left(\frac{\pi\ r_i^2}{T_i . \sqrt{x = 1}} \right)^1 = \frac{\pi\ r_i^2}{T_i} = \text{const}$$

However, in the general case $x \neq 1$ the second Kepler's law will be modified in the following manner [6, 7, 8]: the radius-vector from the central body to the orbital body sweeps out equal areas in equal amount of time with increased ($x > 1$), or decresead ($x < 1$) continuation proportionally to the square root of the mass of the central body which is expressed in units of solar mass, i.e.

$$\left(\frac{\pi\ r_i^2}{T_i . \sqrt{x \neq 1}} \right)^1 = \text{const}$$

The particular case x = 1 relates to the Solar system or to the systems of the stars which have masses near to the mass of the Sun, for example 55 Cancer, etc.

Results for the mean distances r_i and fixed places a_i of the Saturnian moons

The calculation results from the algorithm for the macro parameters of satellites in Saturnian system are presented in Table 1. The Saturnian moons are listed here by orbital period, from shortest to longest. The explanations to the 15 columns of the Table 1 are the following:

In column 1 the serial number of the corresponding Saturnian satellite is given;

In column 2 the satellite name is presented;

In column 3: $\tilde{x}_i$ is the argument of the power function $y = 2^{x-2}$;

In column 4: $\tilde{a}_i$ is the interpolation ordering satellite number, as the 21-th place is the end of the system;

In column 5: $\tilde{d}_i$ is the value of the interpolating function;

In column 6 is presented the formation of approximating linear functional

$l_i(\tilde{f}) = \pi\, x\, \tilde{d}_i$ is the value of the interpolation linear functional;

In column 7: $\tilde{\Delta}_i = (y_i - y_{i-1})$ is the approximating difference;

In column 8: $-\tilde{k}_i = \pi\, x\, \tilde{\Delta}_i$ is subtrahend factor;

In column 9 the formation of the lower and upper interval boundaries of $l_i(\tilde{f})$ are shown. The lower boundary is $\tilde{o}_i = l_i(\tilde{f})$ - $\tilde{k}_i$, as -$\tilde{k}_i$ is the subtrahend factor (column 8);

In column 10: $[\tilde{o}_i,\ l_i(\tilde{f})]$ is the interval of the approximating linear functional;

In column 11: d_i is the function $l_i(f)/(\pi.M_{SAT})$ which we approximate;

In column 12: $l_i(f) = 10\ r_i$ is function which we approach, as $l_i(f)$ is expressed in AU;

In column 13: r_i is the distance (km) between the i-th satellite and the central body Saturn [4, 5];

In column 14: r_i is the distance (AU) between the i-th satellite and the central body Saturn. In the calculations we always express r_i in AU;

In column 15: a_i is the fixed place of the i-th satellite.

Some satellites have the same quantum number, e.g. i = 5, 6, 7, 12 and 13 because of the short distances between their orbits, i.e. the orbits have almost equal major semiaxis. That event may be called *orbital clustering* [12].

On the other hand some quantum numbers i remain not fulfilled (i = 11), which means that there are yet not discovered satellites in this region. The places where they are situated are predicted in the Tables 1 and 2. It should be noticed that the region x_{14} - x_{21} is yet not investigated.

Results for macro parameters of Saturnian satellites on the basis of Kepler's second law

These results of the algorithm calculations are presented in Table 2, as the explanations are the following:

In column 1 a serial number of the corresponding Saturnian satellite is shown;

In column 2 the names of the Saturnian satellites are given;

In column 3 their orbital periods (in years) are presented;

In column 4 the linear interpolation functional is shown

$$l_i(\tilde{f}) = \left(\frac{\pi r_i^2}{T_i \sqrt{x}}\right)^2$$

In column 5 the interpolation function is calculated

$$\tilde{d}_i = l_i(\tilde{f})/(\pi\, M_{SAT})$$

which is an argument in the interpolation functional;

In column 6 the correction $l_i(f) = \pi^2\ r_i$ concerning the functional $l_i(f) = 10\ r_i$ (column 12 in Table 1) is given;

In column 7 the value of

$$d_i = \frac{\pi^2 r_i}{\pi\, M_{SAT}} = \frac{\pi r_i}{2.858\times10^{-4}} = 10992.277 r_i \text{ is}$$

presented.

Results for the accuracy

In Table 3 are shown the statistical data for the degree of approximation of $l_i(\tilde{f}) = l_i(f)$, respectivly $\tilde{d}_i = d_i$, for each Saturnian satellite.

In column 1 is calculated the accuracy (%) of $l_i(\tilde{f})$ (column 4 in Table 2) versus $l_i(f) = 10\ r_i$ (column 12 in Table 1);

In column 2 is calculated the accuracy (%) of $l_i(\tilde{f})$ (column 4 in Table 2) versus $l_i(f) = \pi^2\ r_i$ (column 6 in Table 2);

In column 3 is calculated the accuracy (%) of $\tilde{d}_i$ (column 5 in Table 2) versus di (column 7 in Table 2).

Conclusion

New model for the distances and fixed places of the moons in Saturnian system is proposed in the present paper. This model is presented as an algorithm in Tables 1 and 2. It describes well the observed macrostructure and macro parameters of the 61 satellites in the Saturnian system.

In this relation the presented improved algorithm for the Saturnian-centric distances and fixed places of the moons in Saturnian system is more precise and possesses the corresponding physical meaning.

From Tables 1 and 2 is seen that Saturn has two groups of satellites. The regular satellites (i = 5-10) move along nearly circular orbits in the planet's orbital plane, revolving about it in the same sense as the planet spins. In contrast, the so-called irregular satellites (i = 12, 13 …) are generally smaller in size and are characterized by large orbits with significant eccentricity, inclination or both. The differences in their characteristics suggest that the regular (i = 5-10) and irregular (i = 12, 13 …) satellites are formed by different mechanisms: the regular satellites are believed to have formed in an accretion disk around the planet, like a miniature Solar System, whereas the irregulars are generally thought to be captured planetesimals. Most of the irregular moons are collisional remnants of larger satellites that were fragmented after capture, rather than being captured independently [12].

It seems also possible to investigate the distances of Saturn rings from the planet. There are some difficulties with the rings of Saturn, which are very extended in the equatorial plane and distances from the planet are not well defined [3]. However our model has the advantage, that he gives also the intervals of the distances, hence, we can have information about the width of the rings. We will investigate in the future not only the planetary satellites, but also the planetary rings. These studies are important for the physics of the whole Saturnian system.

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