Space Plasma Physics

Some modern analyses of space plasma waves

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Why space plasma waves are important?

- Carrier of information (remote-sensing):
  gives information on the source and the medium
  where the wave propagates.
- Carrier of momentum flux, energy flux, etc :
  e.g., solar wind acceleration
- Main ingredient of various instabilities
- Efficient scatterer of energetic particles: e.g. cosmic ray diffusion and acceleration.
- Sources of anomalous dissipation
- Provides opportunities to test nonlinear wave theories: ideal natural laboratory

… and a lot more…

What can we do from the space plasma wave data?

- wave frequency (spacecraft frame)
- mode identification
- polarization, ellipticity, helicity, etc
- power spectrum in the time domain

…

- dispersion relation (k and w)
- higher order spectrum in time domain
  (bi-coherence, tri-coherence etc)
- higher order spectrum in spatial domain
- phase statistics

…

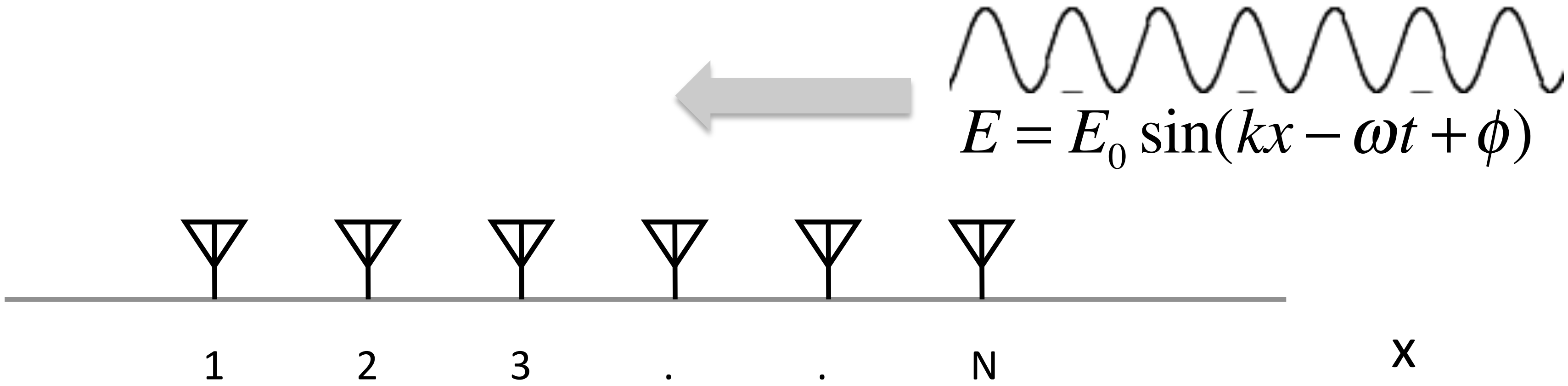
improvement of data quality
(accuracy, time resolution)
multi-point measurement

Multi-point measurement

Determination of the wave number

Determination of the wave frequency is straight-forward, since # data points is very large.

In contrast, # data points for space is usually small.



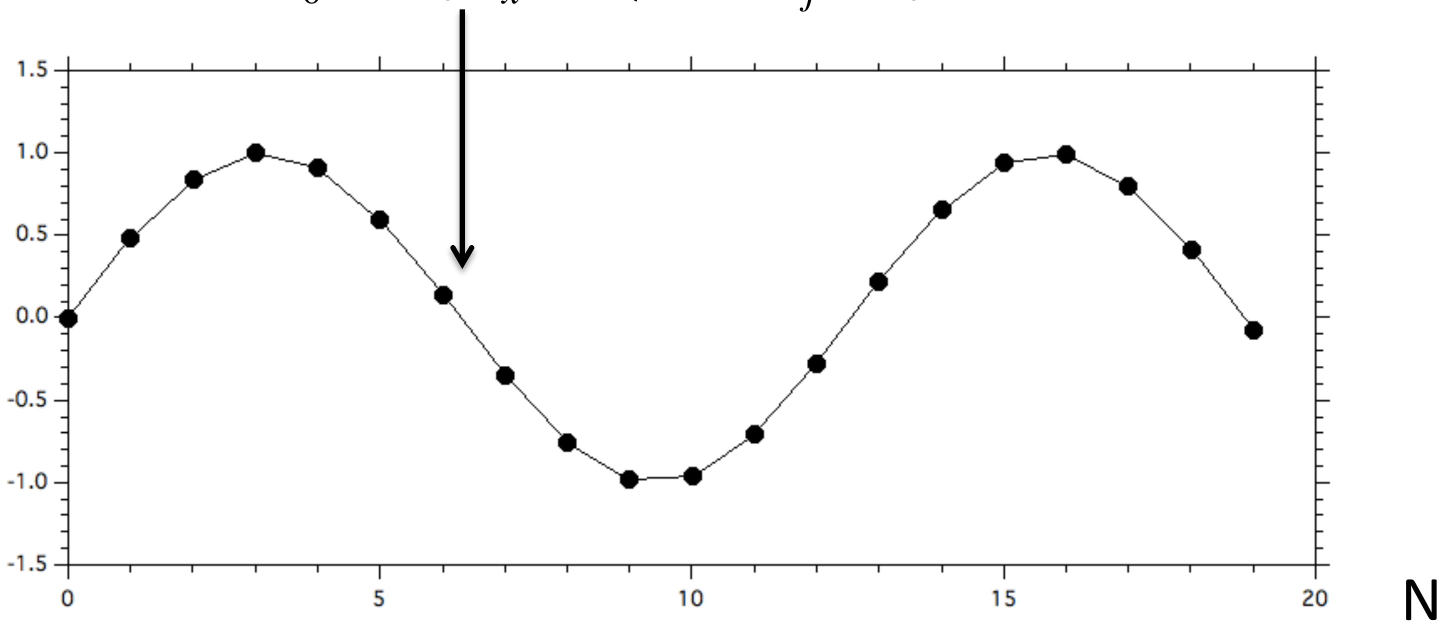
$$E = E_0 \sin(kx - \omega t + \phi)$$

Suppose that a simple sinusoidal wave propagates into a region where N observation points are aligned.

Let the position of the obs points be x=d, 2d, …, Nd, then the measurements are:

$$E_0 \sin(k_x d + \phi), \quad E_0 \sin(2k_x d + \phi), \quad \ldots \quad E_0 \sin(Nk_x d + \phi)$$

$$= E_0 \sin(jk_x d + \phi) \equiv E_j, \quad j = 1 \ldots N$$



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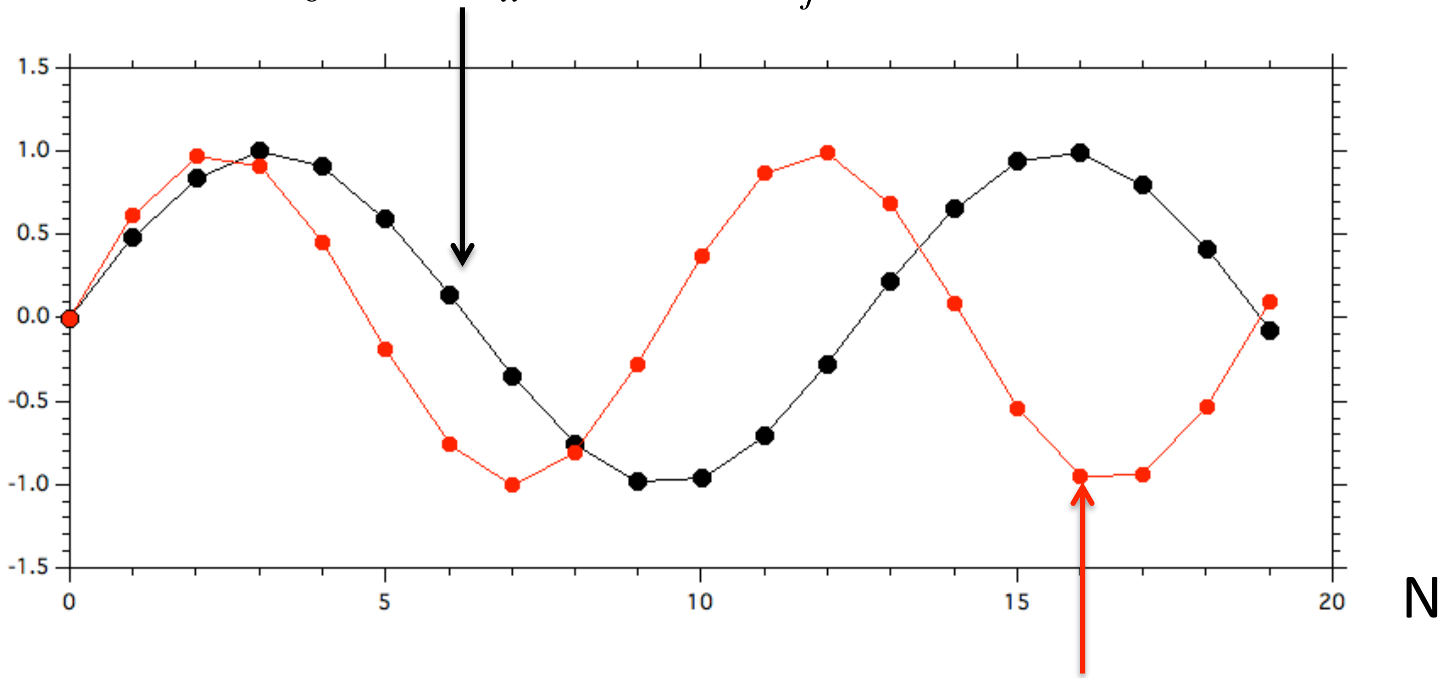


Now multiply by the test function, $f(K) = \sin(jKd)$ and sum over j=1 to N.

What we just did is the Fourier analysis.
In a more general framework, we should compute:

$$S(K) = \sum_{j=1}^{N} E_j \sin(jKd), \quad C(K) = \sum_{j=1}^{N} E_j \cos(jKd)$$

$$P(K) \equiv (S(K)^2 + C(K)^2)$$

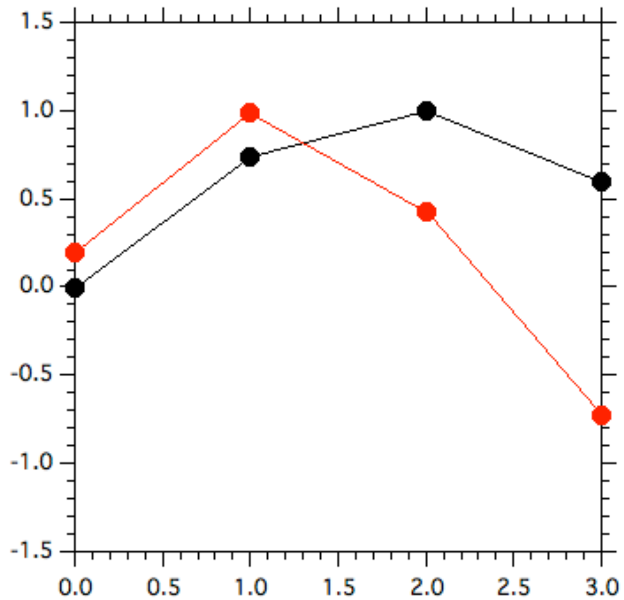
The Fourier power should be maximized at

$$K = k$$

(estimation of the wave number)

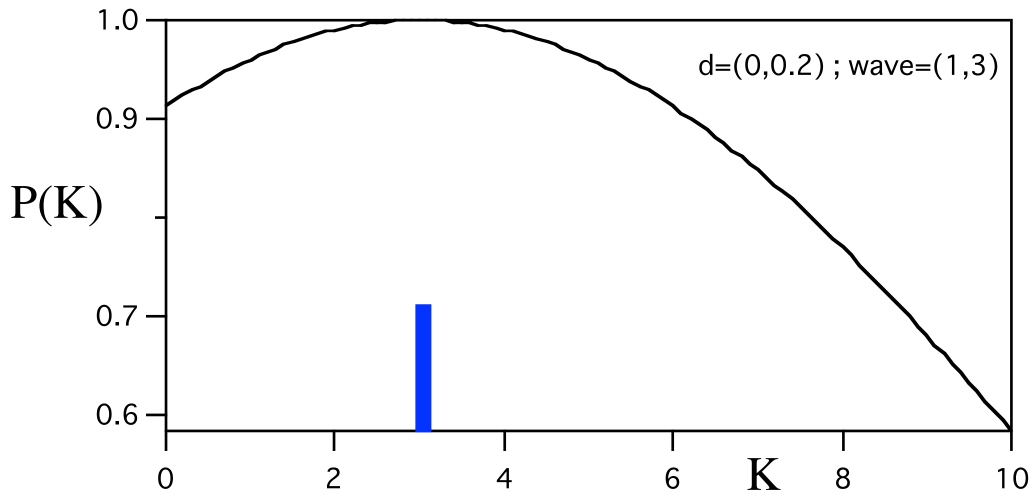
Is this true?

true, but when N is a small number (which usually is the case for spacecraft experiments), this method is useless.

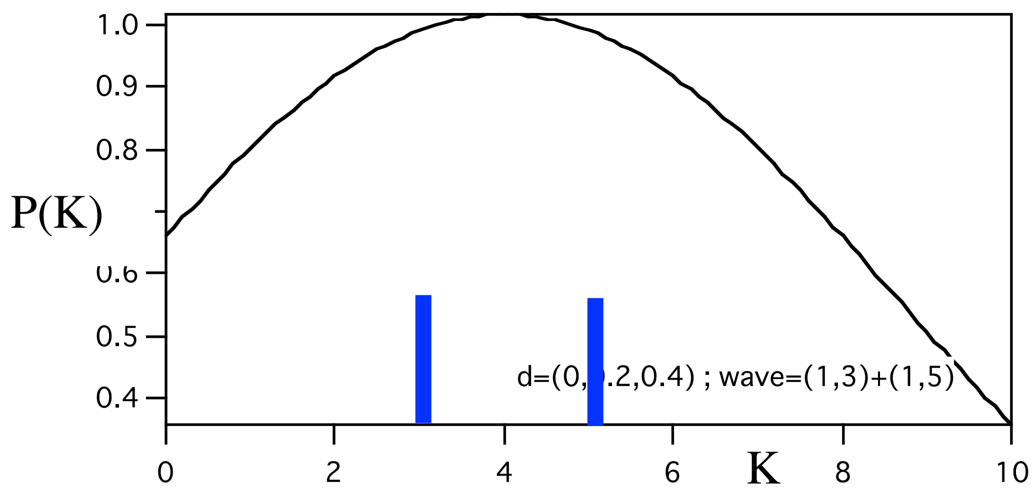


N=4

Determination of k: examples



(upper panel)
#S/C= 2 (at x=0, 0.2)
#waves=1 (k=3)



(lower panel)
#waves=2 (k=3, 5)
#S/C= 3 (at 0, 0.2, 0.4)

- Low resolution
- Two waves are not separated even though #S/C=#waves+1
- aliasing (if kd>2π)

Linear transformation

$$y_n = \sum_{k}^{N} a_{k,n} u_k$$

output kernel input

- If we choose

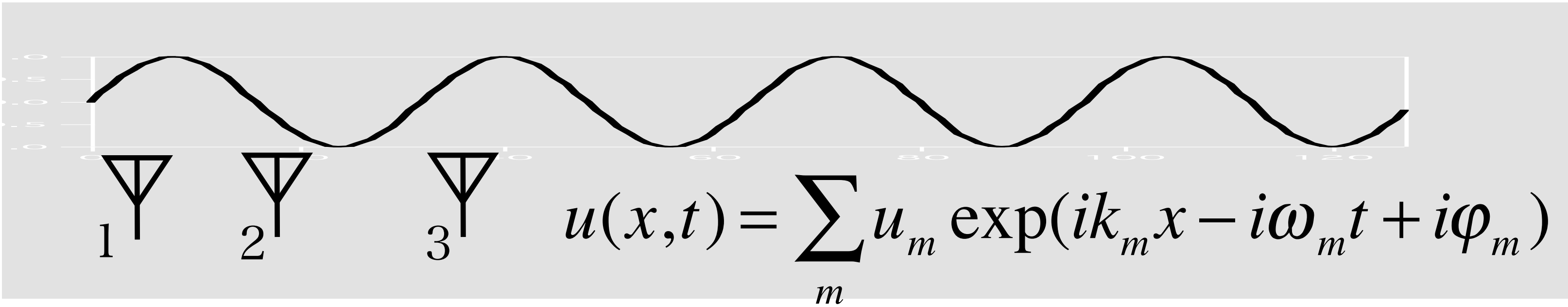
$$a_{k,n} = e^{2\pi ikn/N}$$

then we have the Fourier transform. But this does not work if N is a small number.

- Instead, try to find a better kernel

$$a'_{k,n}$$

making use of the input value.



$$u(x,t) = \sum_m u_m \exp(ik_m x - i\omega_m t + i\varphi_m)$$

Signal
$$\mathbf{u} = \sum_m \{\hat{u}_m \exp(ik_m x_1 + i\phi_m), \hat{u}_m \exp(ik_m x_2 + i\phi_m), \ldots\}$$
$$= \{u_1, u_2, \ldots\}$$

'Naive' kernel

$$\mathbf{a}(K) = \{\exp(-iKx_1), \exp(-iKx_2), \ldots\} = \{a_1, a_2, \ldots\}$$

'Naive' power spectrum （BeamForming)

$$P(K) = <|\mathbf{a}\cdot\mathbf{u}|^2> = a_i^* < u_i^* u_j > a_j = \mathbf{a}^+ \cdot \mathbf{R} \cdot \mathbf{a}$$

where $R_{ij} = < u_i u_j >$

ensemble average

Power density matrix

One can determine a better **a’** by requiring

- P(K) is minimized when **a**’ is used
- constraint, |**a . a**’ |=1

This problem can be solved by Lagrange’s undetermined coefficient method.

$$P(K) = <|\mathbf{a}'\cdot\mathbf{u}|^2> = a'^*_i < u^*_i u_j > a'_j$$

Define $$F = a'^*_i < u^*_i u_j > a'_j - \lambda(a^*_i a'_i a_j a'^*_j - 1)$$

and require $$\frac{\partial F}{\partial a_i} = \frac{\partial F}{\partial a^*_i} = \frac{\partial F}{\partial \lambda} = 0$$

Solution:

Weighted kernel (Capon)

$$\mathbf{a}'(K) = \frac{\mathbf{R}^{-1}}{\mathbf{a}^{+}\mathbf{R}^{-1}\mathbf{a}} \cdot \mathbf{a}$$

This gives a

Better estimate of power

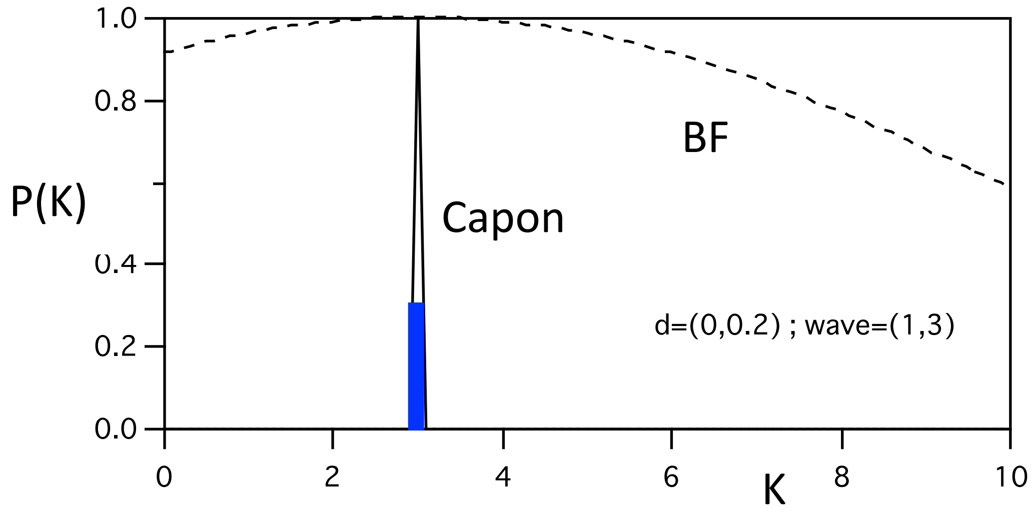
$$P(K) = \frac{1}{\mathbf{a}^{+}\mathbf{R}^{-1}\mathbf{a}}$$

Capon, IEEE, 1969

Motschmann et al, 1996; Glassmeier et al.,2001:'The wave telescope'

Pinçon and Lefeuvre, 1991:'The k-filtering'

Determination of k: examples



(upper panel)
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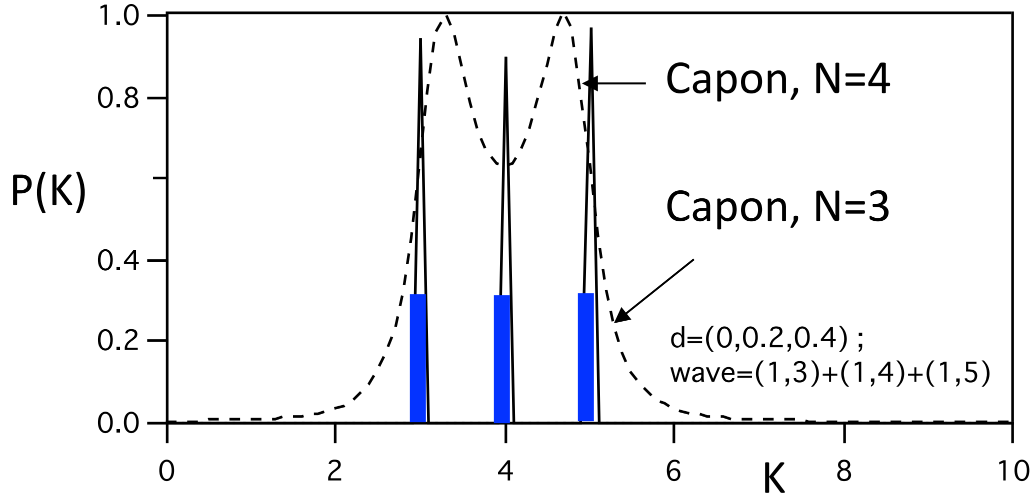
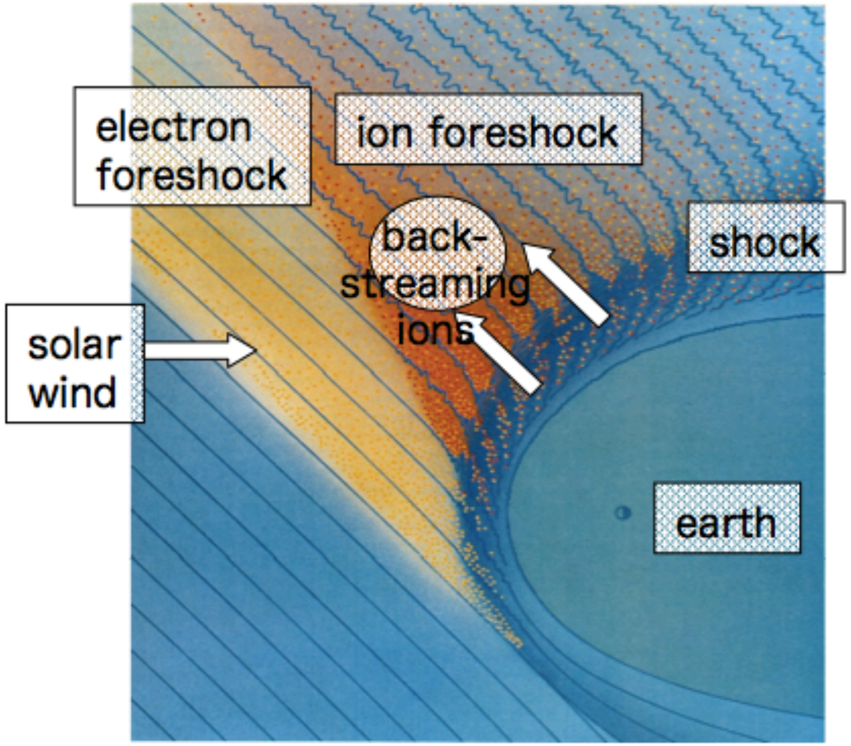
#waves=1 (k=3)

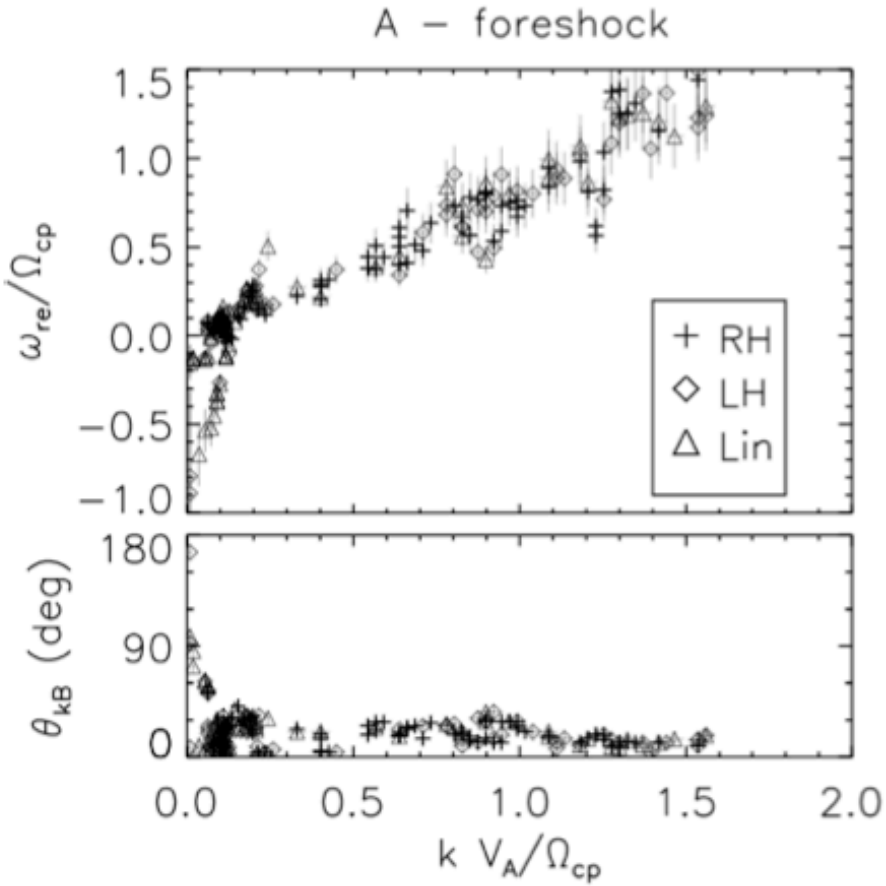


(lower panel)
#waves=3 (k=3, 4, 5)
#S/C= 3 (at 0, 0.2, 0.4)
#S/C= 4 (at 0, 0.2, 0.4, 0.6)

- Resolution improved.
-Need N+1 s/c to resolve superposition of N waves.

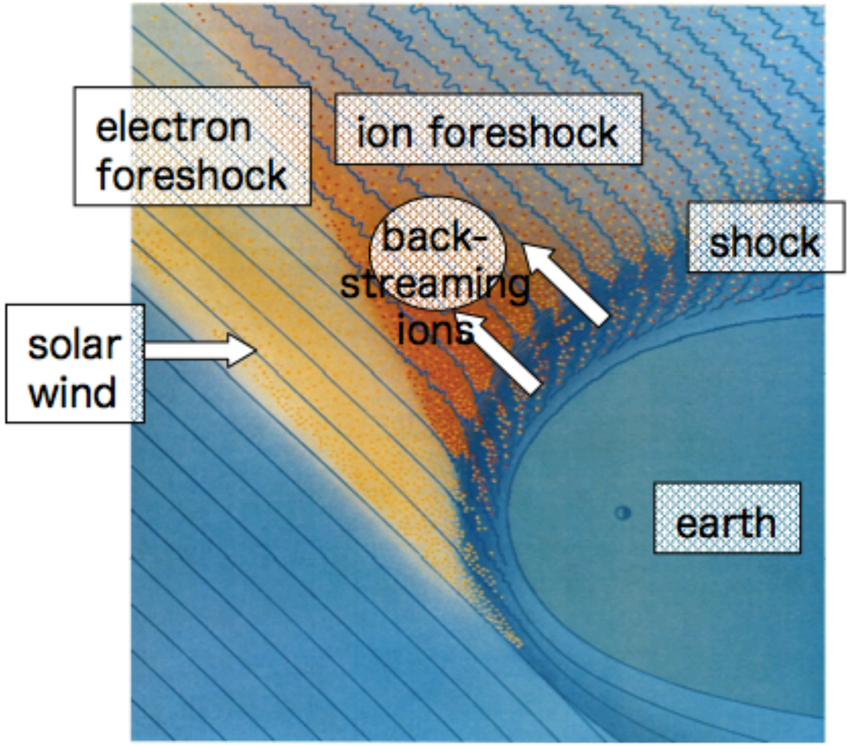
Dispersion relation of foreshock waves



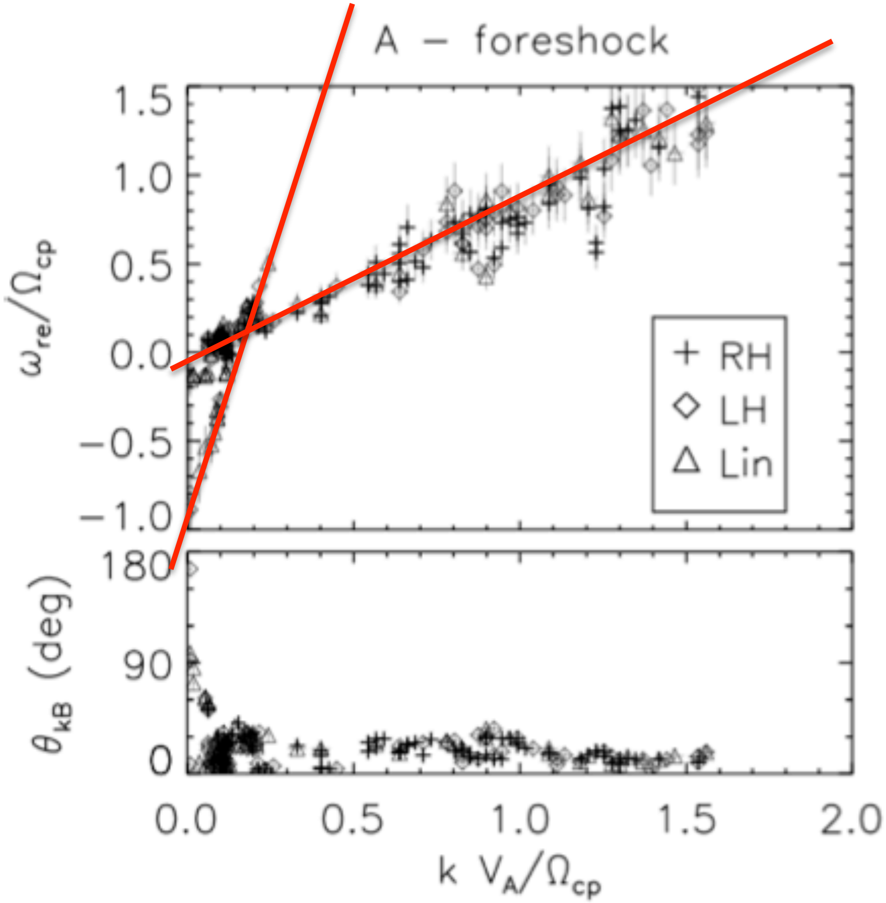


Narita and Glassmeier, 2005

Dispersion relation of foreshock waves



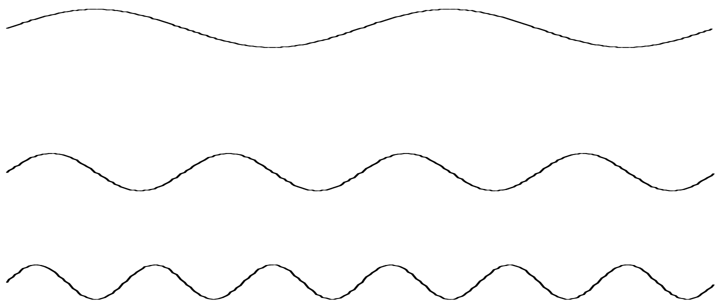
$$\omega - kv_b = \Omega_i \qquad \omega = kv_A$$



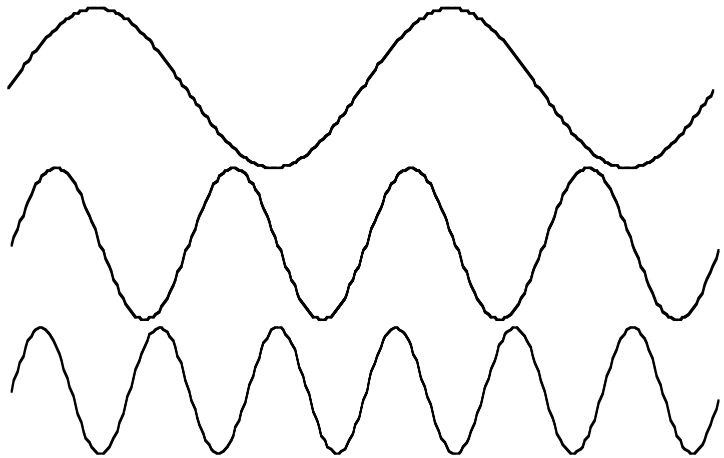
Narita and Glassmeier, 2005

Bi-spectrum

Linear and nonlinear waves



- small amplitude
- superposition principle

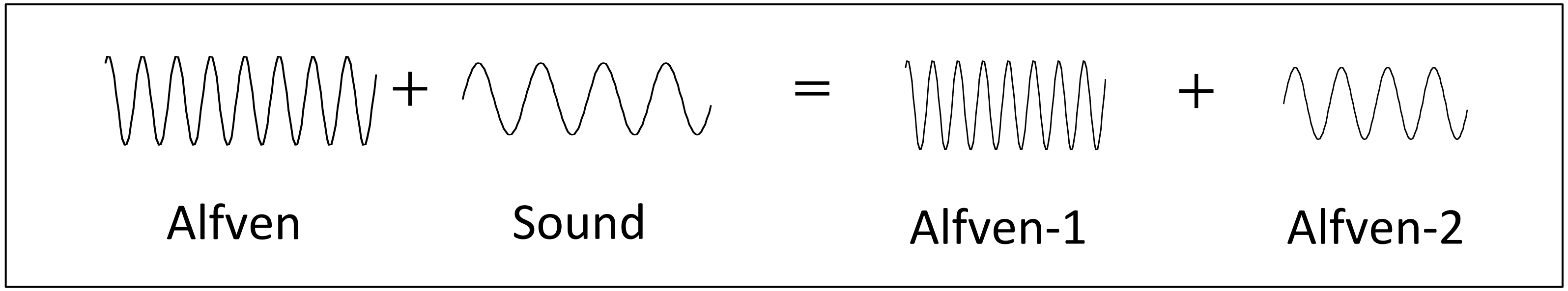


- finite amplitude
- background medium modified
  by presence of the waves

Interaction of finite amplitude waves

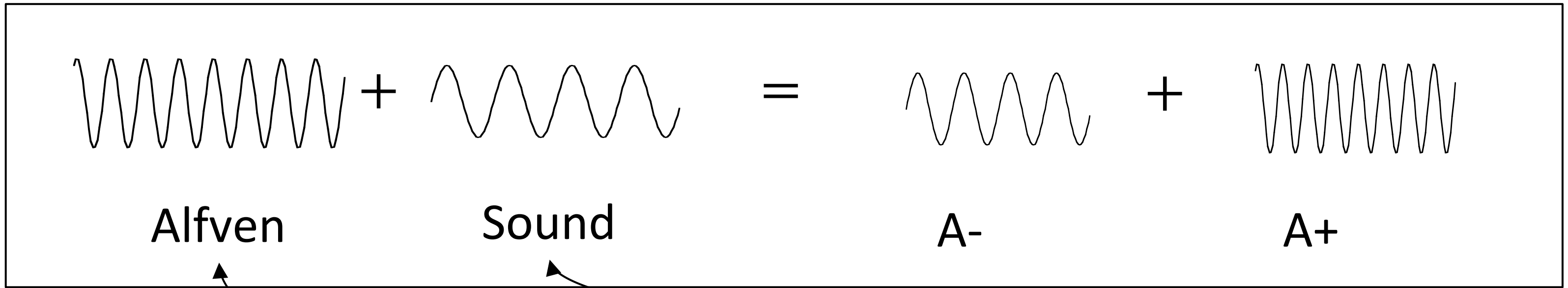
- Large amplitude MHD waves are ubiquitous in space.
- One should be able to see “Nonlinear interaction among waves”.

For example



- The interaction can be captured by Higher-Order statistics.
- Evaluation both in ‘time’ and ‘spatial’ domains.
- # of data points for the ‘spatial’ domain very limited.

Interaction of finite amplitude waves



$$B = B_0 + \delta B \exp i\Phi_1 \qquad n = n_0 + \delta n \exp i\Phi_2 \qquad \Phi_i = k_i x - \omega_i t + \varphi_i$$

$$Bn = B_0 n_0 + \underbrace{\delta B \exp i\Phi_1 + \delta n \exp i\Phi_2}_{\text{Linear}} + \underbrace{\delta B \delta n \exp i\Phi_3}_{\text{Nonlinear}}$$

Nonlinear coupling produces oscillations with $\Phi_1 + \Phi_2, \Phi_1 - \Phi_2$

Resonance: $\omega_1 + \omega_2 = \omega_3 \quad k_1 + k_2 = k_3 \quad \varphi_1 + \varphi_2 = \varphi_3 + c$

Matching of frequency, wavenumber, and the phase.
If the phase is random, the coupling vanishes via averaging.

Bi-spectrum

In time domain

$$F(\omega_1,\omega_2) =< b(\omega_1)n(\omega_2)b^*(\omega_1+\omega_2) >$$

In spatial domain

$$F(k_1,k_2) = < b(k_1)n(k_2)b^*(k_1+k_2) >$$

- If the relative phase of b1, n2, b3* is fixed at a certain value, the ensemble average gives a finite value.
- Otherwise, i.e., if these waves just happen to exist but they are not nonlinearly interacting, the ensemble average is cancelled.

Evaluation of bi-spectrum in timedomain

Bi-spectrum by BeamForming

Signal $\mathbf{u} = \{\hat{u}\exp(ikx_1),...,\hat{u}\exp(-ikx_M)\}$

'Naive' kernel $\mathbf{a}(K) = \{\exp(-iKx_1),...,\exp(-iKx_M)\}$

$$p < k_1, k_2 > = < \mathbf{a}(k_1)\cdot\mathbf{u}\ \mathbf{a}(k_2)\cdot\mathbf{v}\ (\mathbf{a}(k_1+k_2)\cdot\mathbf{u})^* >$$

Evaluation of bi-spectrum in spatial domain

Bi-spectrum by Capon

Narita et al, 2009

Signal $\mathbf{u} = \{\hat{u}\exp(ikx_1),\ldots,\hat{u}\exp(-ikx_M)\}$

Evaluate $R_{ij} = < u_i^* u_j >$

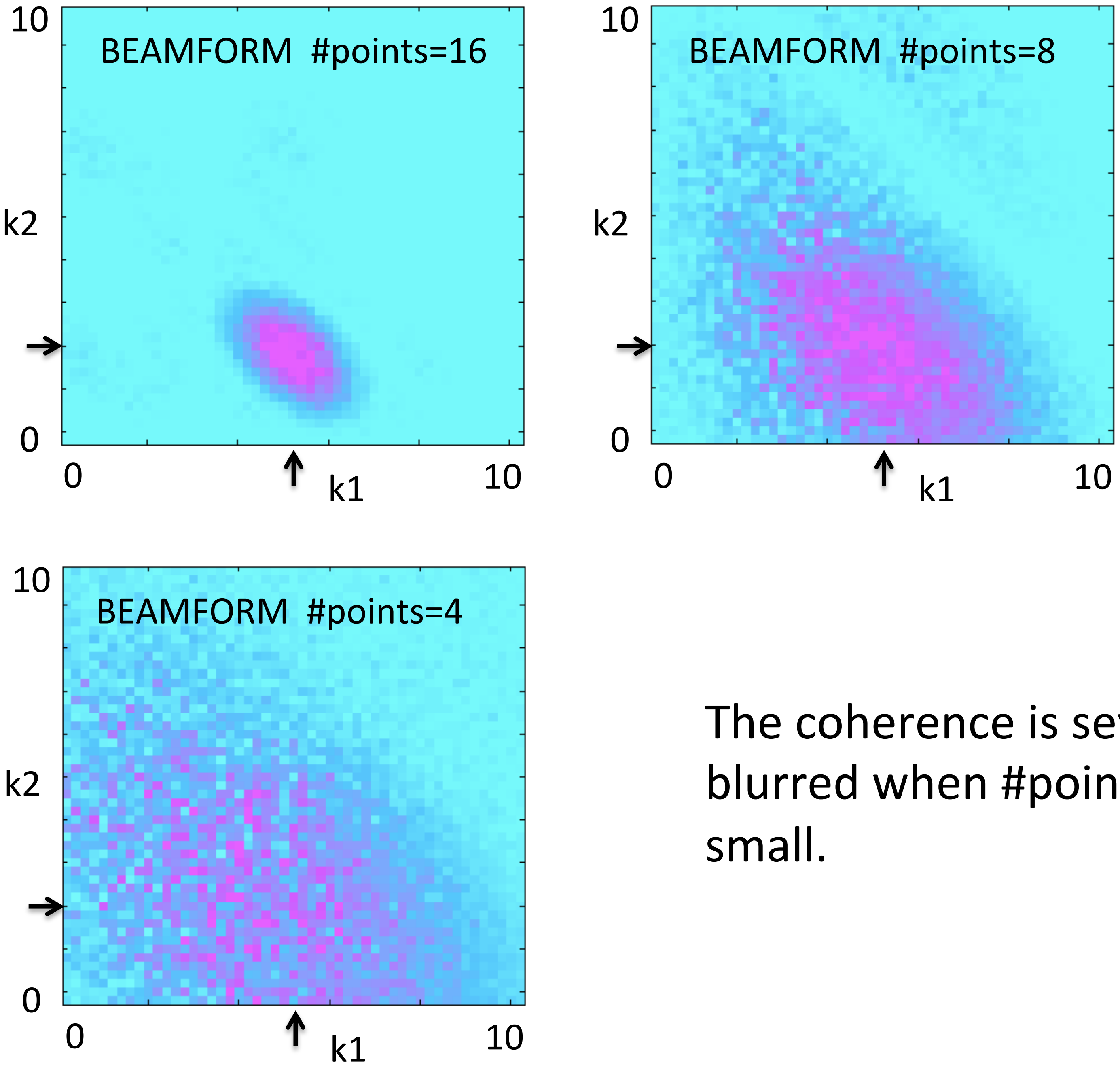
Define capon kernel $\mathbf{a}'(K) = \dfrac{\mathbf{R}^{-1}}{\mathbf{a}^{+}\mathbf{R}^{-1}\mathbf{a}} \cdot \mathbf{a}$

$$p < k_1, k_2 > = < \mathbf{a}'(k_1)\cdot\mathbf{u}\ \mathbf{a}'(k_2)\cdot\mathbf{v}\ (\mathbf{a}'(k_1+k_2)\cdot\mathbf{u})^* >$$

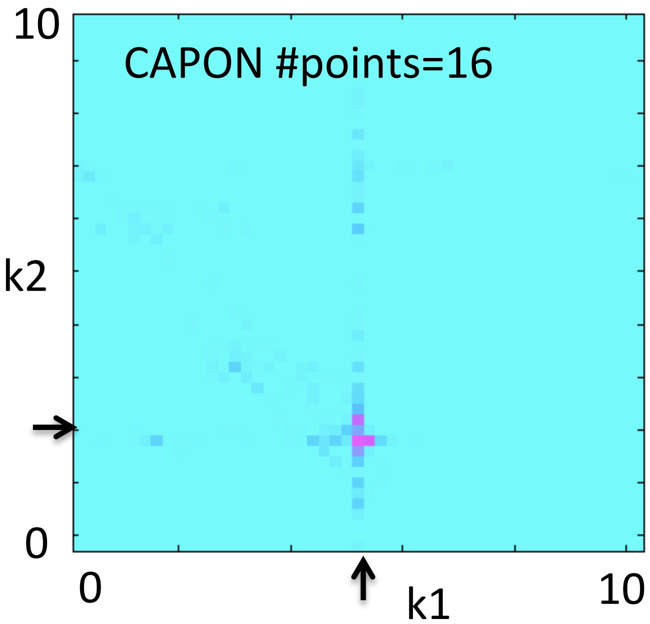
Evaluation of bi-spectrum: examples

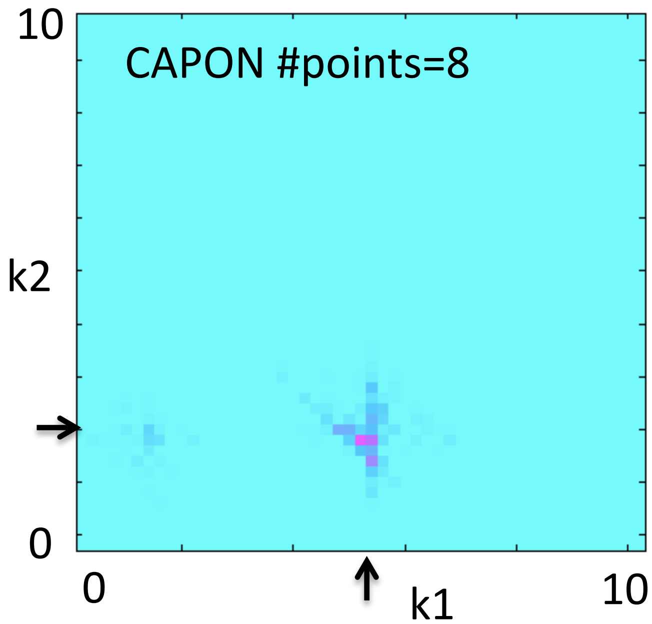
Waves with k1=5, k2=2, and k3=7 are in resonance.
Plus random noise.

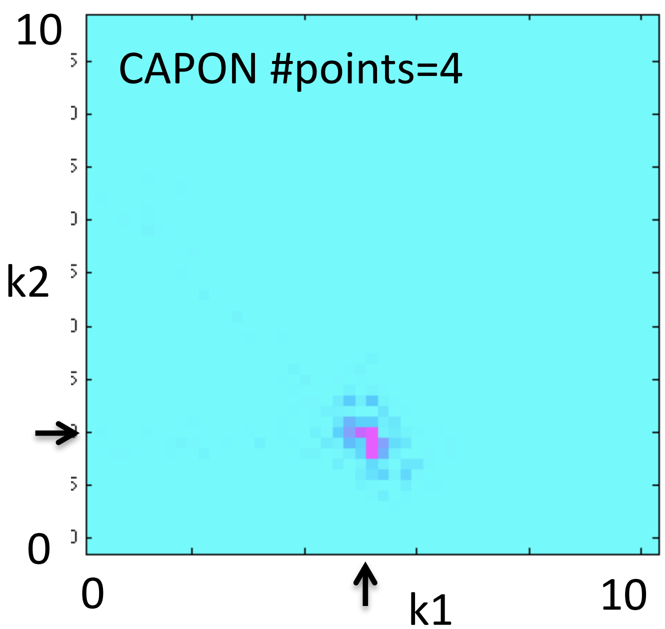
Compare the bi-coherence due to the BF and the capon.
Vary # of data points (= # of spacecraft)



The coherence is severely blurred when #points is small.

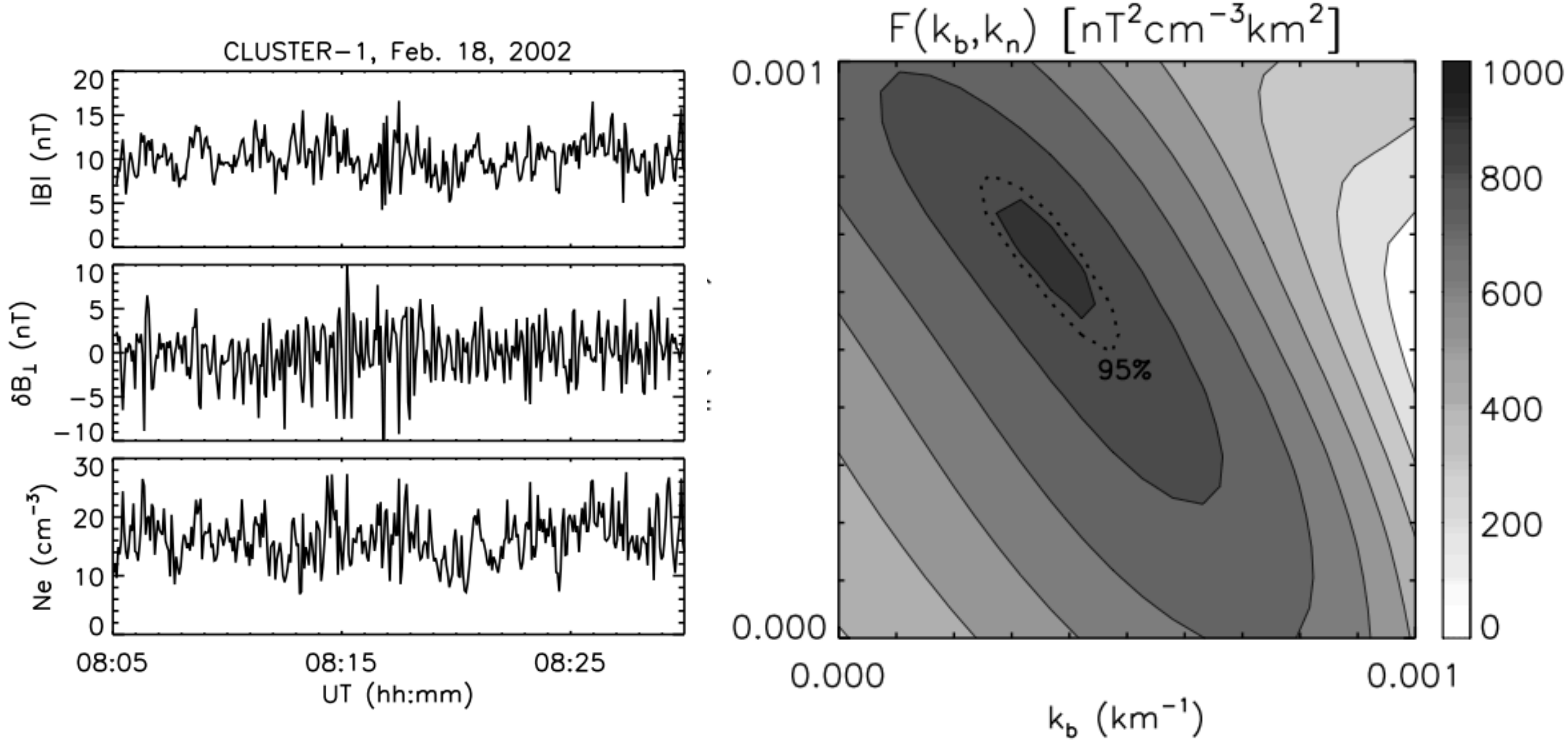






Four S/C is enough!

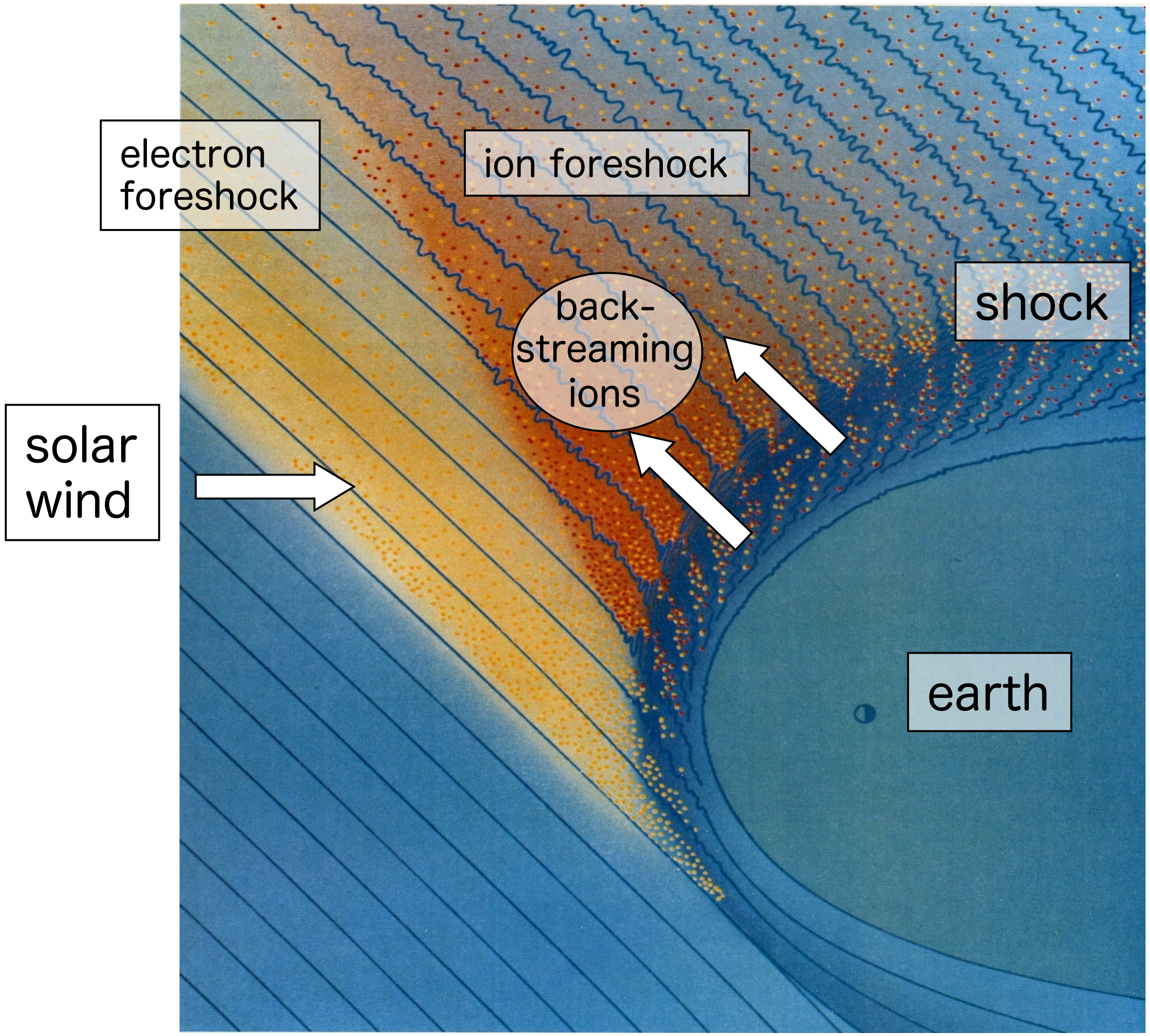
(Three is fine, too, if one uses B and n data)



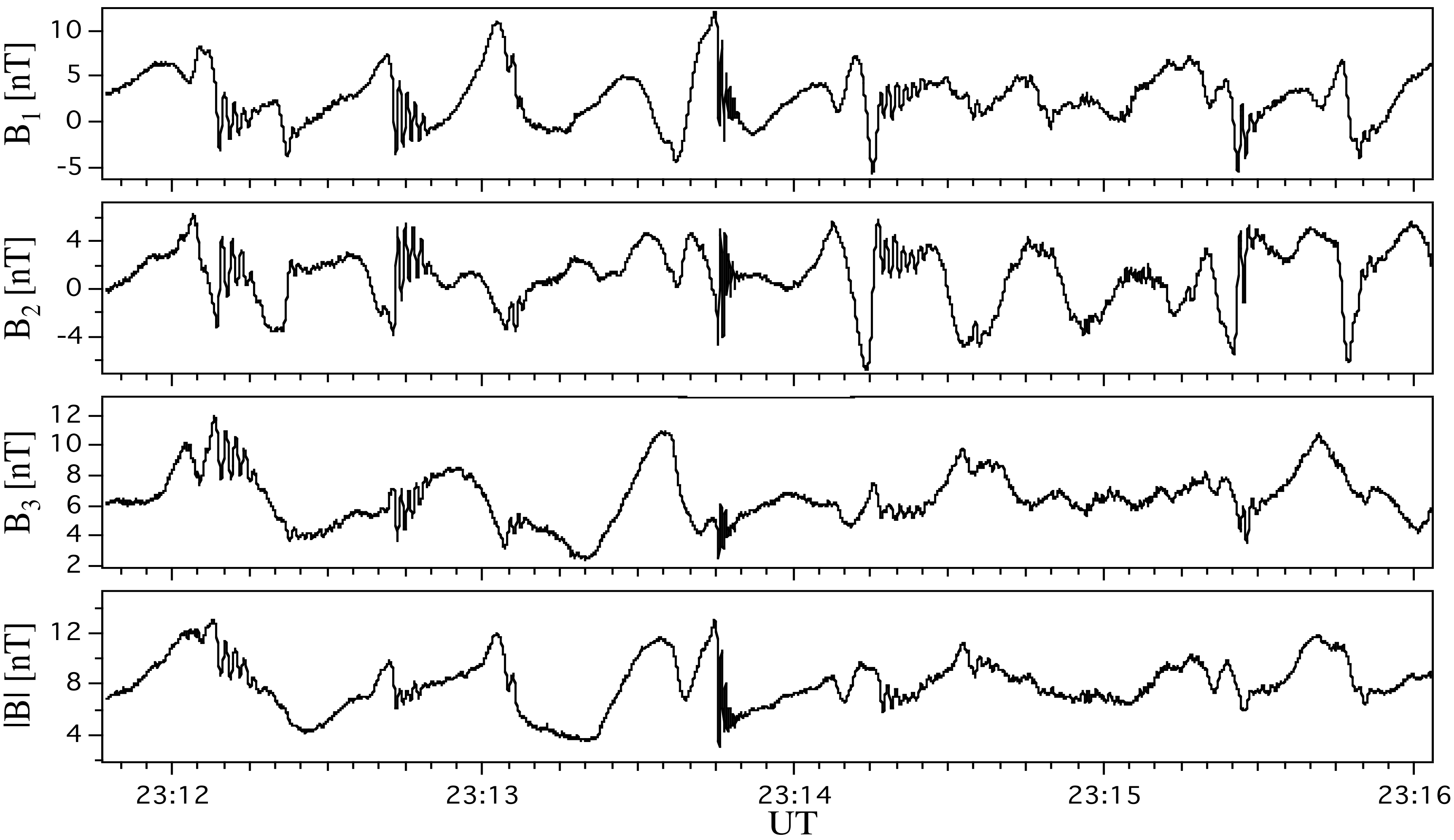
Narita et al., 2009

Phase statistics

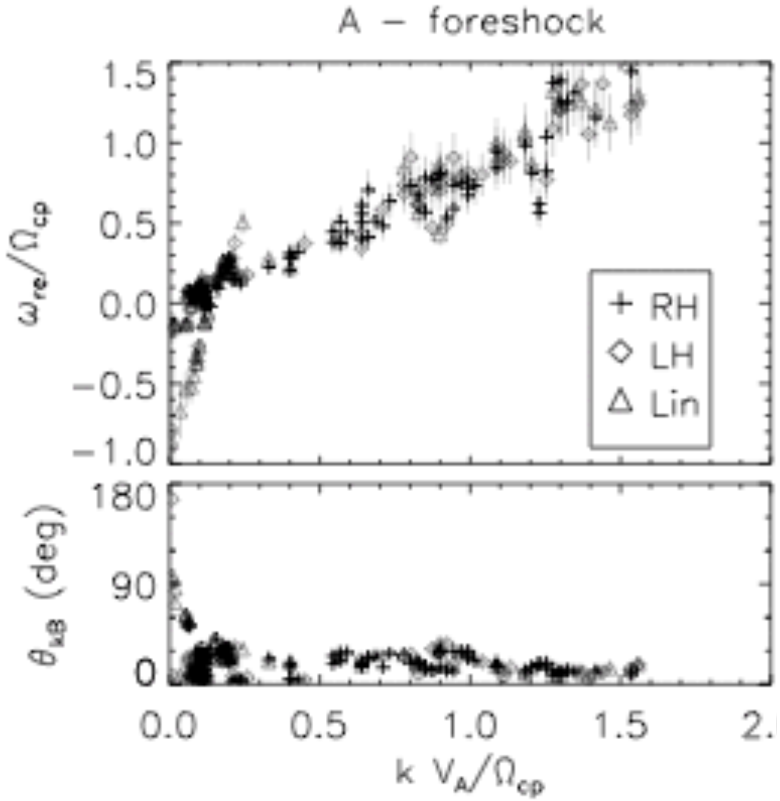
The earth's foreshock



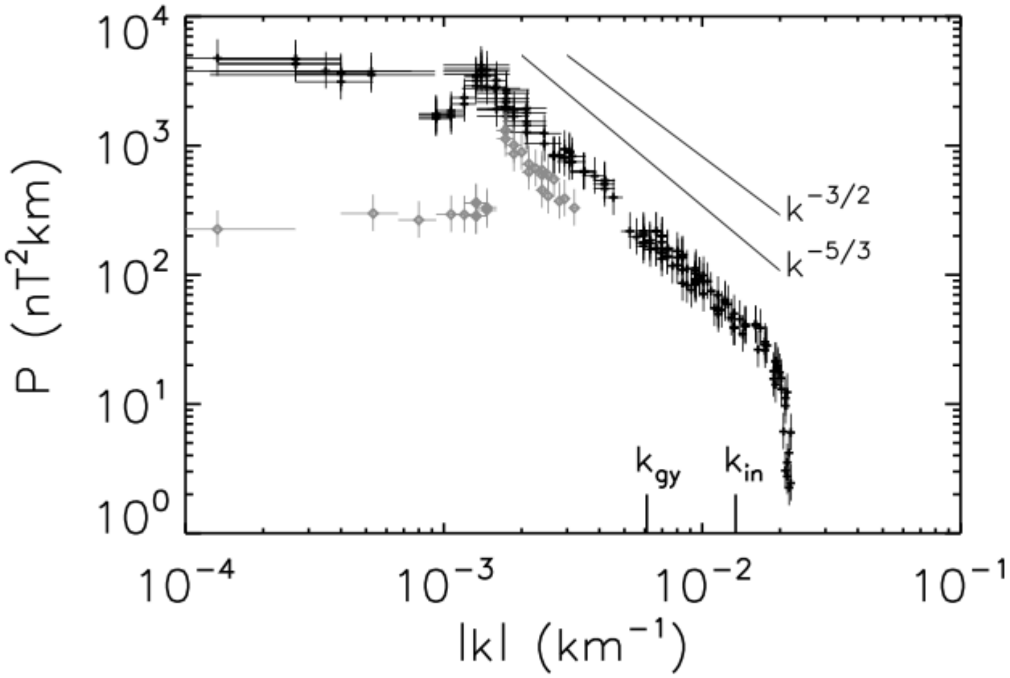
Large amplitude waves in the earth's foreshock



Dispersion relation and the spectrum



Narita and Glassmeier, 2005



Narita et al, 2005

(ideal) MHD equations

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho \mathbf{u})=0$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u^2+p-\frac{\mathbf{B}_\perp^2}{2})=0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v})+\frac{\partial}{\partial x}(\rho u \mathbf{v}-\mathbf{B}_\perp)=0$$

$$\frac{\partial}{\partial t}(\frac{\rho(u^2+\mathbf{v}^2)}{2}+\frac{\mathbf{B}_\perp^2}{2}+\frac{P}{\gamma-1})$$

$$+\frac{\partial}{\partial x}(\frac{\rho u}{2}(u^2+\mathbf{v}^2)+u\mathbf{B}_\perp^2-\mathbf{v}\cdot\mathbf{B}_\perp+\frac{\gamma P u}{\gamma-1})=0$$

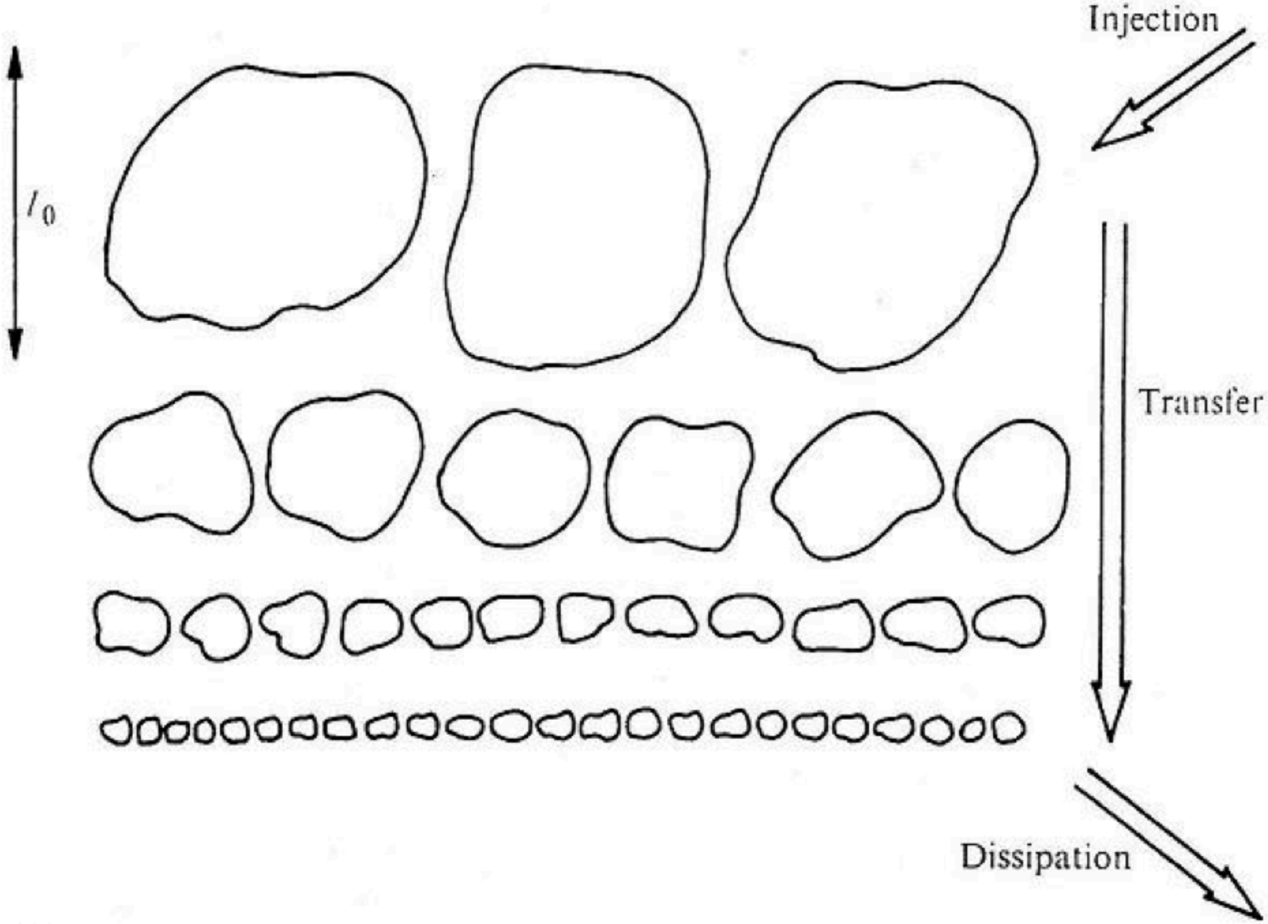
$$\frac{\partial \mathbf{B}_\perp}{\partial t}+\frac{\partial}{\partial x}(u\mathbf{B}_\perp-\mathbf{v}B_x)=0$$

$$\frac{\partial B_x}{\partial x}=0$$

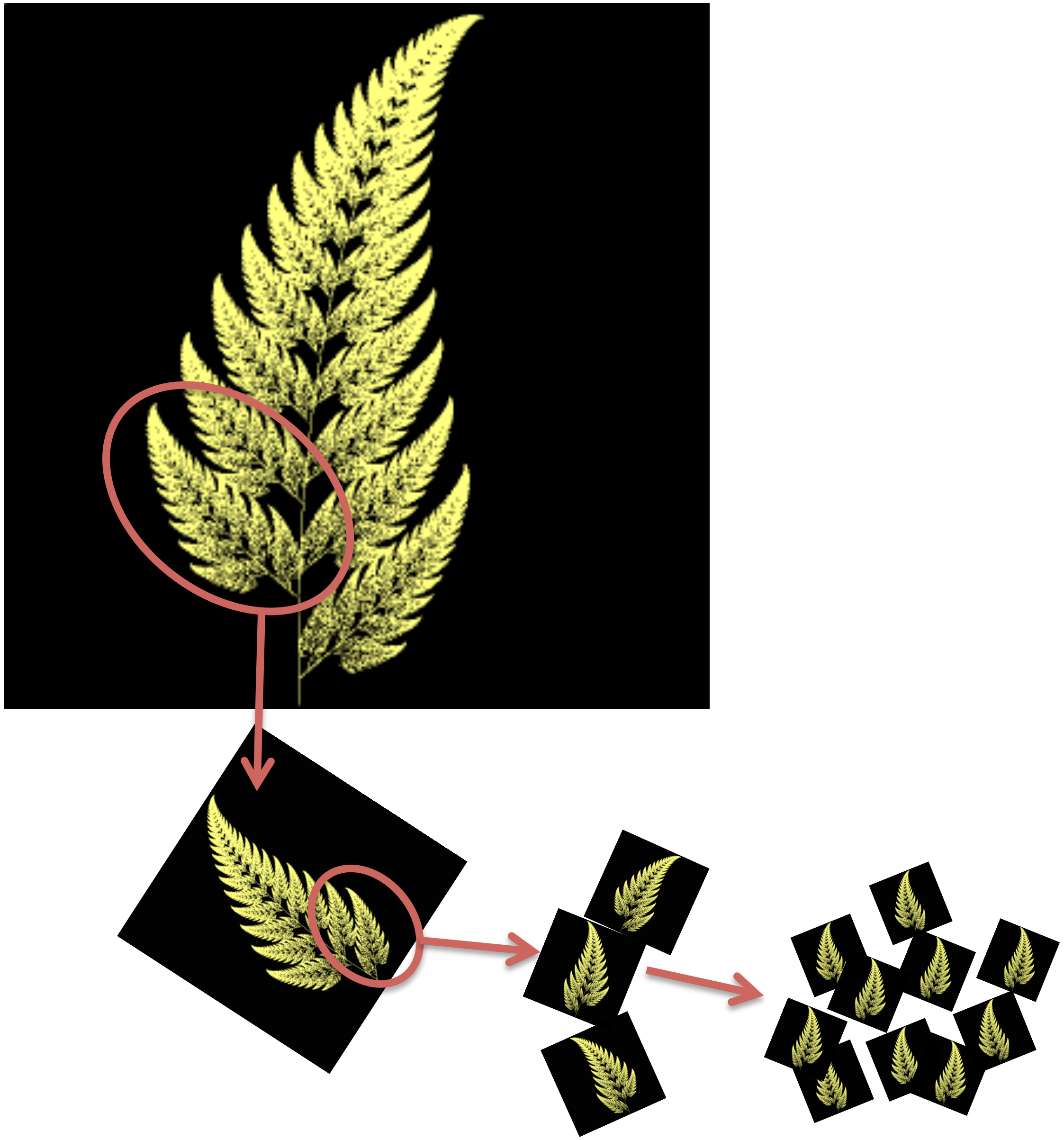
invariant under

$$t\to\alpha t\,,\,x\to\alpha x$$

Cascade of vortices

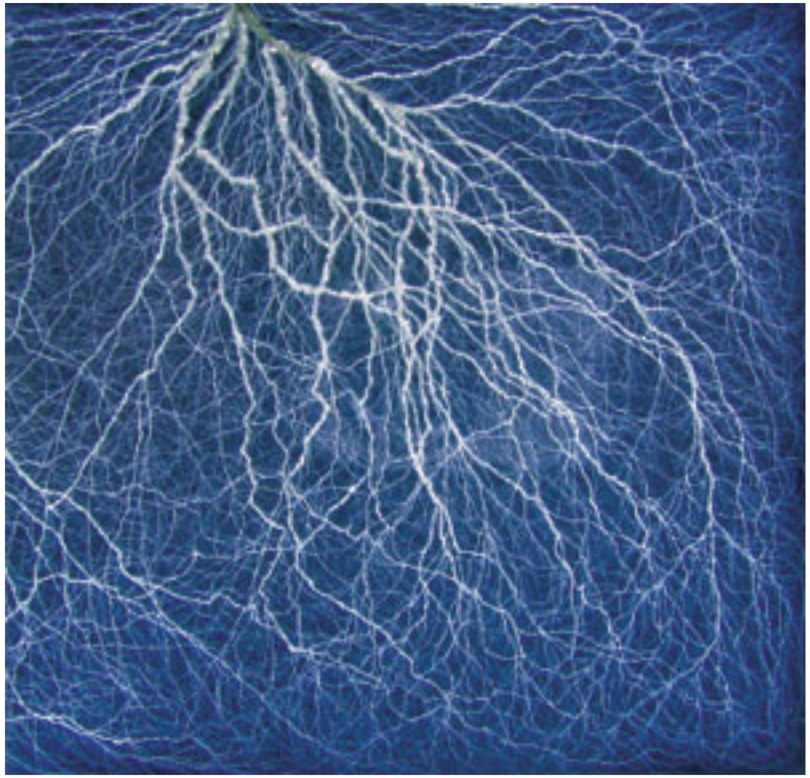


Fern: the fractal



Cloud and Lightning : fractals

from wikipedia



http://www.um.u-tokyo.ac.jp/publish_db/2006jiku_design/sano.html

Roles of the phases

Fern (real & simulation) and cloud/lightning have similar power-law type spectrum.
But they look very different in real space.

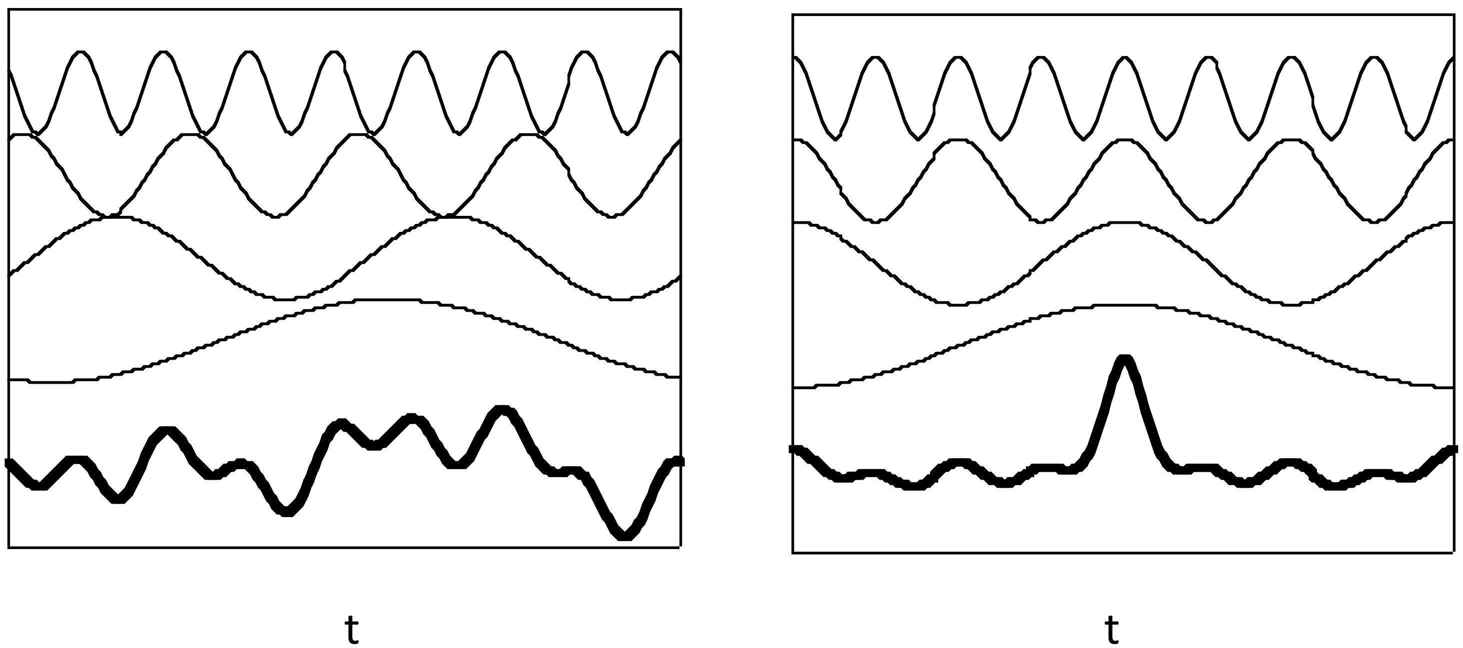
why?

-- phase distribution

Superposition of waves with different phase distributions

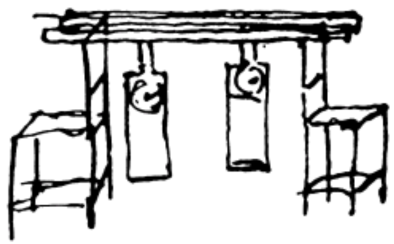
waves with random phase

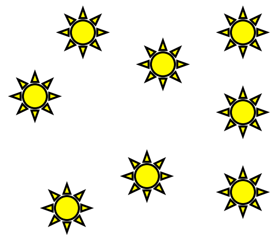
waves with random phase



Phase coherence in nonlinear systems

- Manifestation of nonlinearity
  Examples: Huygens pendulums / Fireflies / ...





- Are the foreshock MHD waves phase coherent?

Phase coherence in MHD waves

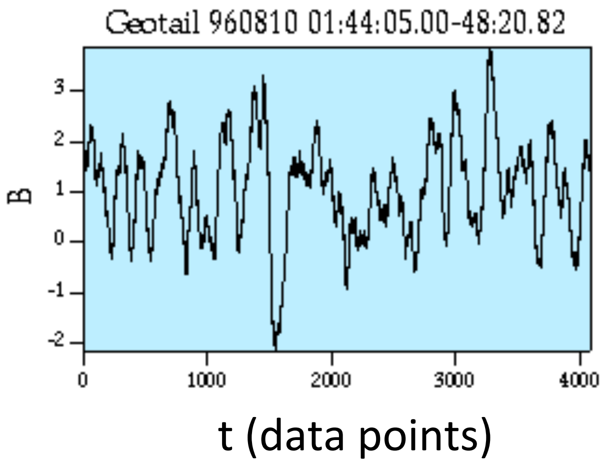
- Manifestation of nonlinearity
  - *Huygens pendulums / Fireflies / etc*
- Assumed in quasi-linear theories
  - *Random Phase Approximation almost always assumed*
  - *Particle diffusion processes can be qualitatively different in phase correlated turbulence*
- Finite phase coherence exits in MHD turbulence in space
  - *Evaluated using Geotail magnetic field data*

Fourier transform: amplitude, frequency, and <u>phase</u>

$$b(t) = \sum_{\omega} \hat{b}(\omega) \sin(\omega t + \phi_{\omega})$$

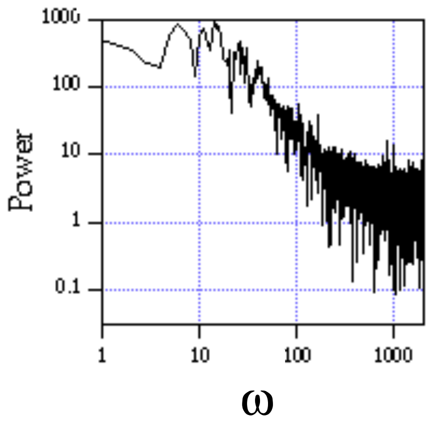
Time series

$b(t)$



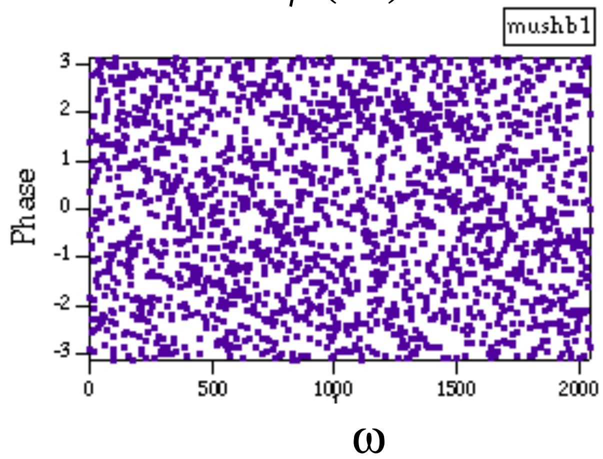
Power spectrum

$|\hat{b}(\omega)|^2$



Phase distribution

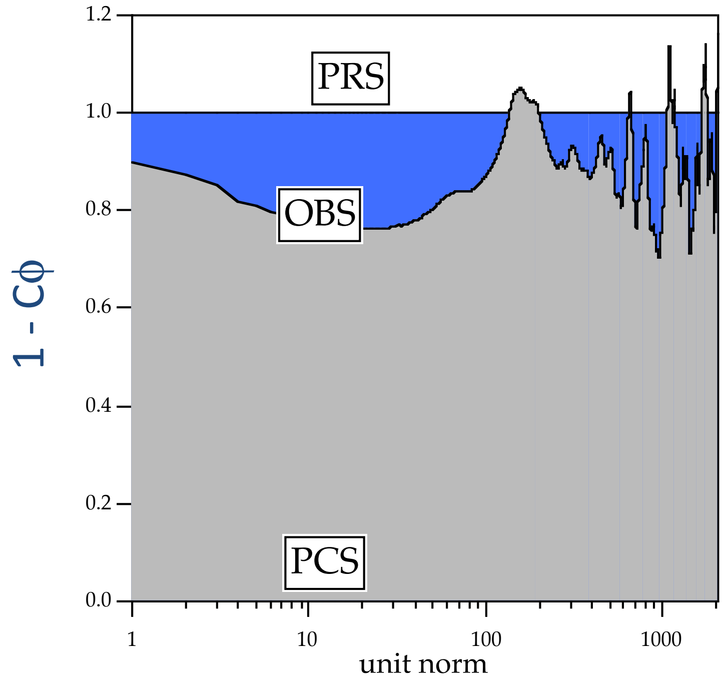
$\phi(\omega)$



Surrogate data analysis

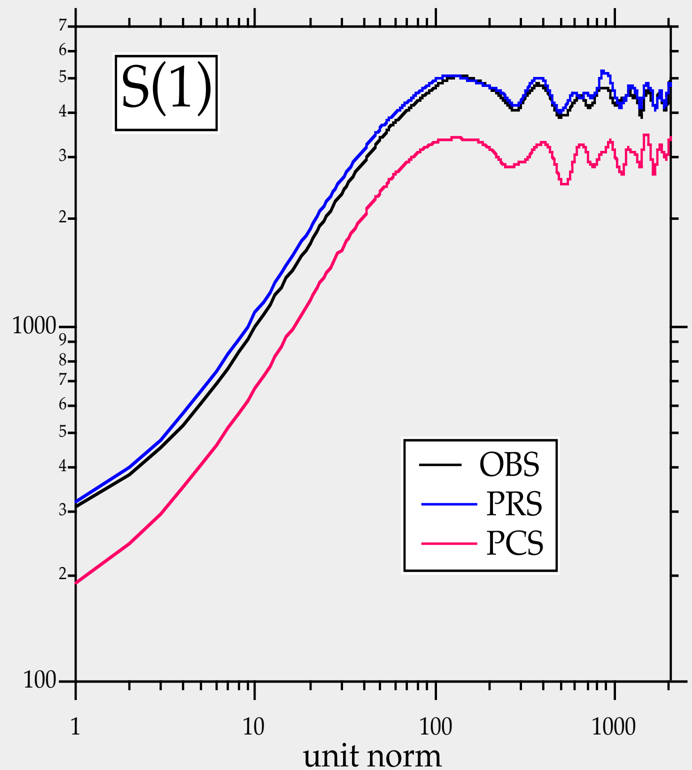
Define “phase coherence index”

$$C_\phi = \frac{S_{PRS} - S_{OBS}}{S_{PRS} - S_{PCS}}$$



Evaluate the Structure function S(1)

$$S(m,\tau) = \sum_t | b(t+\tau) - b(t) |^m$$



Define the phase correlation in REAL space

- From the original time series (OBS), make
  - Phase Randomized Surrogate (PRS) and
  - Phase Correlated Surrogate (PCS)

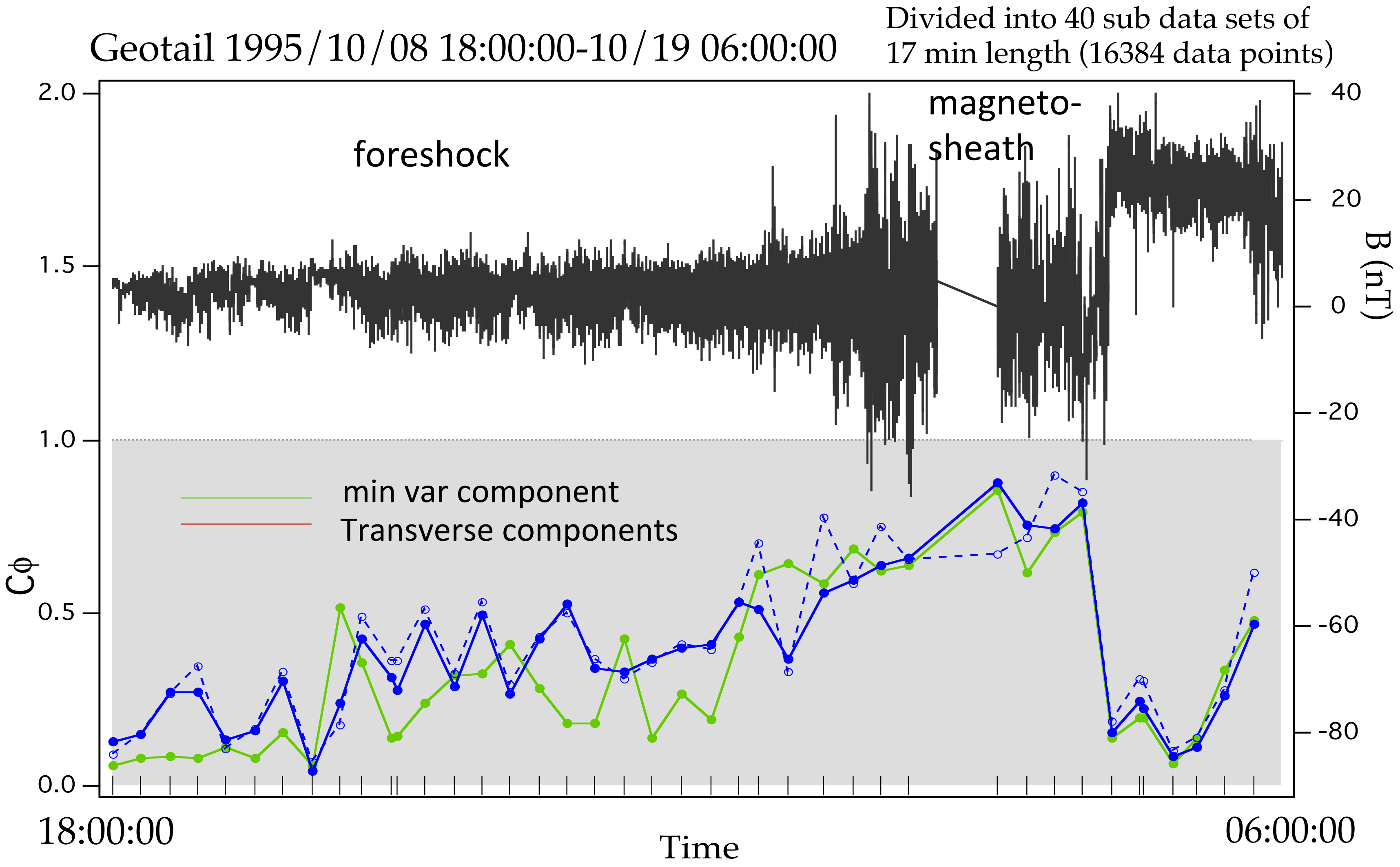
- Evaluate Structure functions for each dataset

$$S(m,\tau) = \sum_t | b(t+\tau) - b(t) |^m$$

- Define the phase coherence index

$$C_\phi = \frac{S_{PRS} - S_{OBS}}{S_{PRS} - S_{PCS}}$$

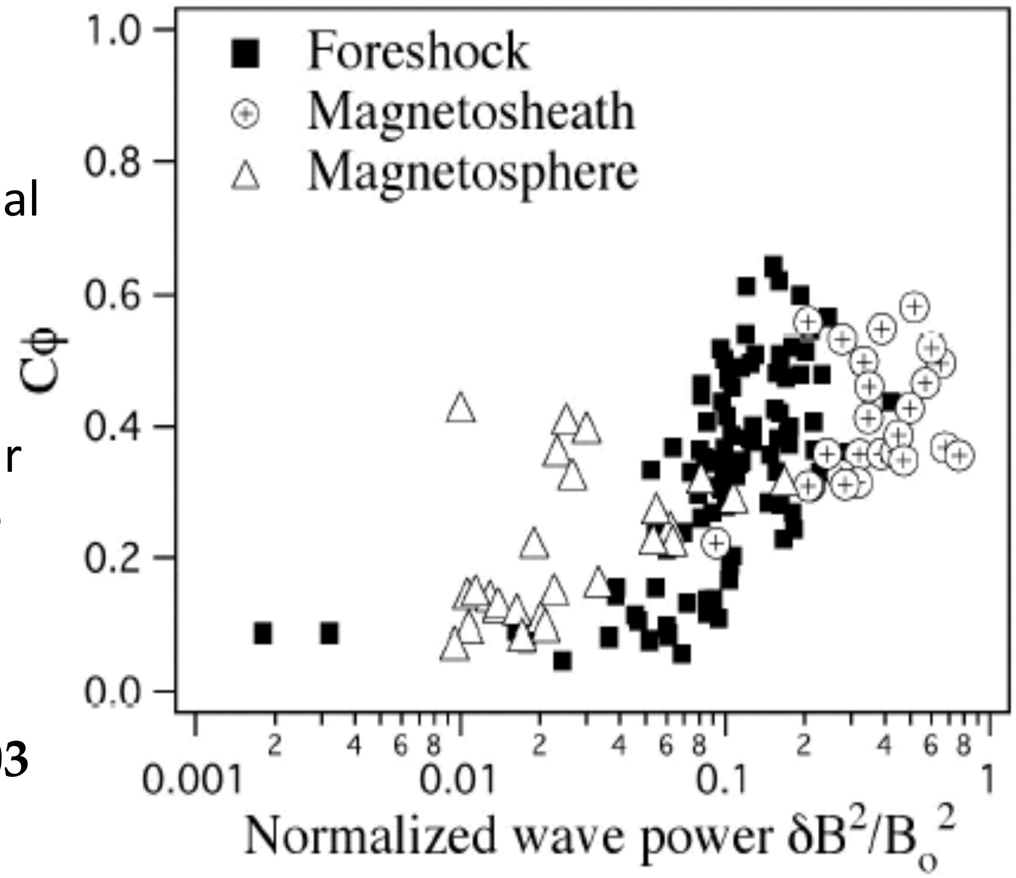
Evolution of Cϕ as S/C travels through various regions



Wave amplitude dependence of Cϕ

Phase coherence index Cϕ evaluated by comparing structure functions of the original and surrogate datasets.

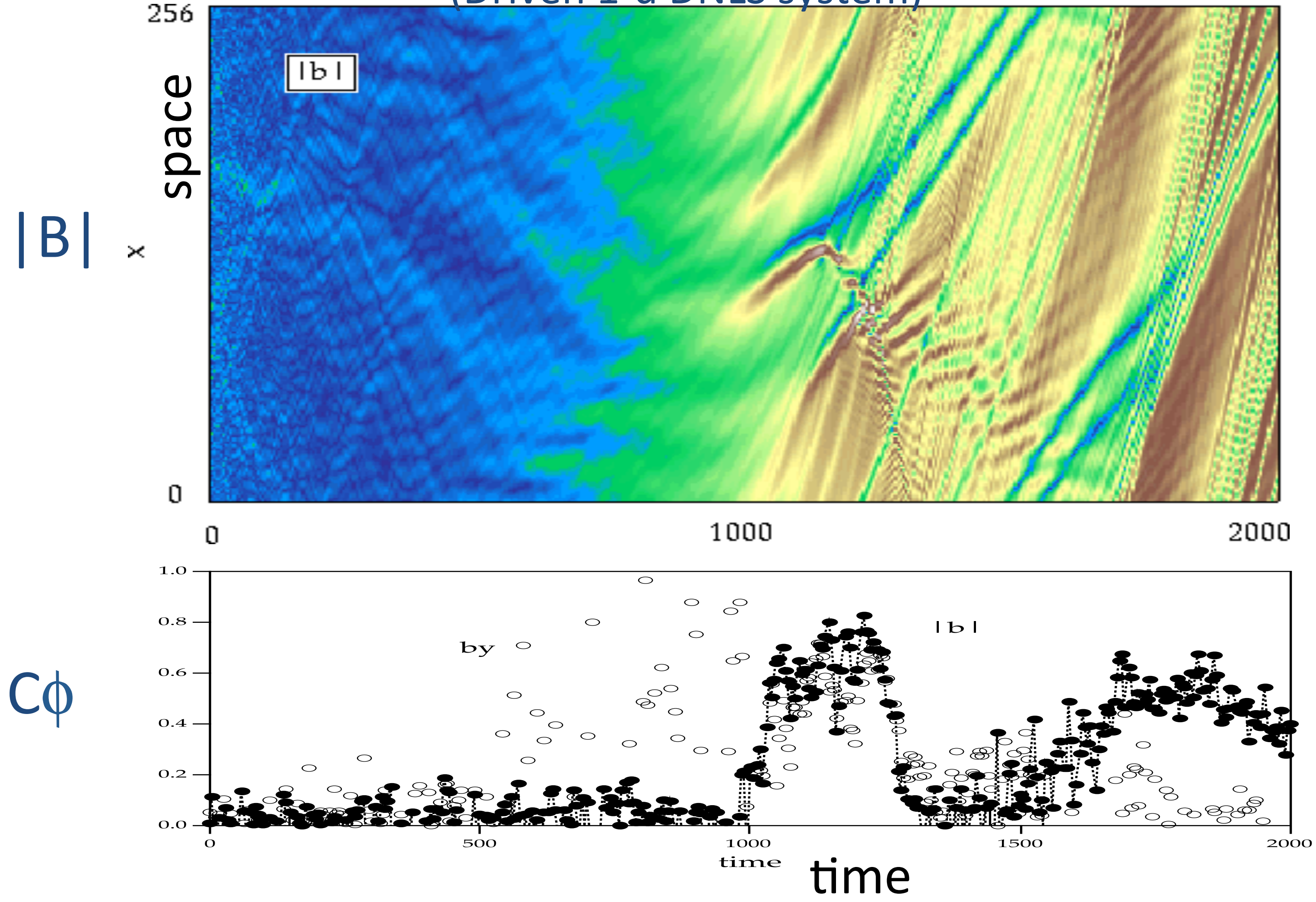
Positive correlation with the turbulence amplitude: Nonlinear interaction generates the phase coherence



Hada, Koga, Yamamoto, 2003
Koga and Hada, 2003

Application to numerical simulation data

(Driven 1-d DNLS system)



Summary

- Waves in space plasma provides various opportunities to evaluate linear and nonlinear processes taking place in the plasma.
- Concepts and some examples of multi-point measurement, bi-spectra in time and spatial domains, and phase statistics are introduced.
- Effort to construct a method to drag as much information as possible from given dataset is an extremely important subject of science.

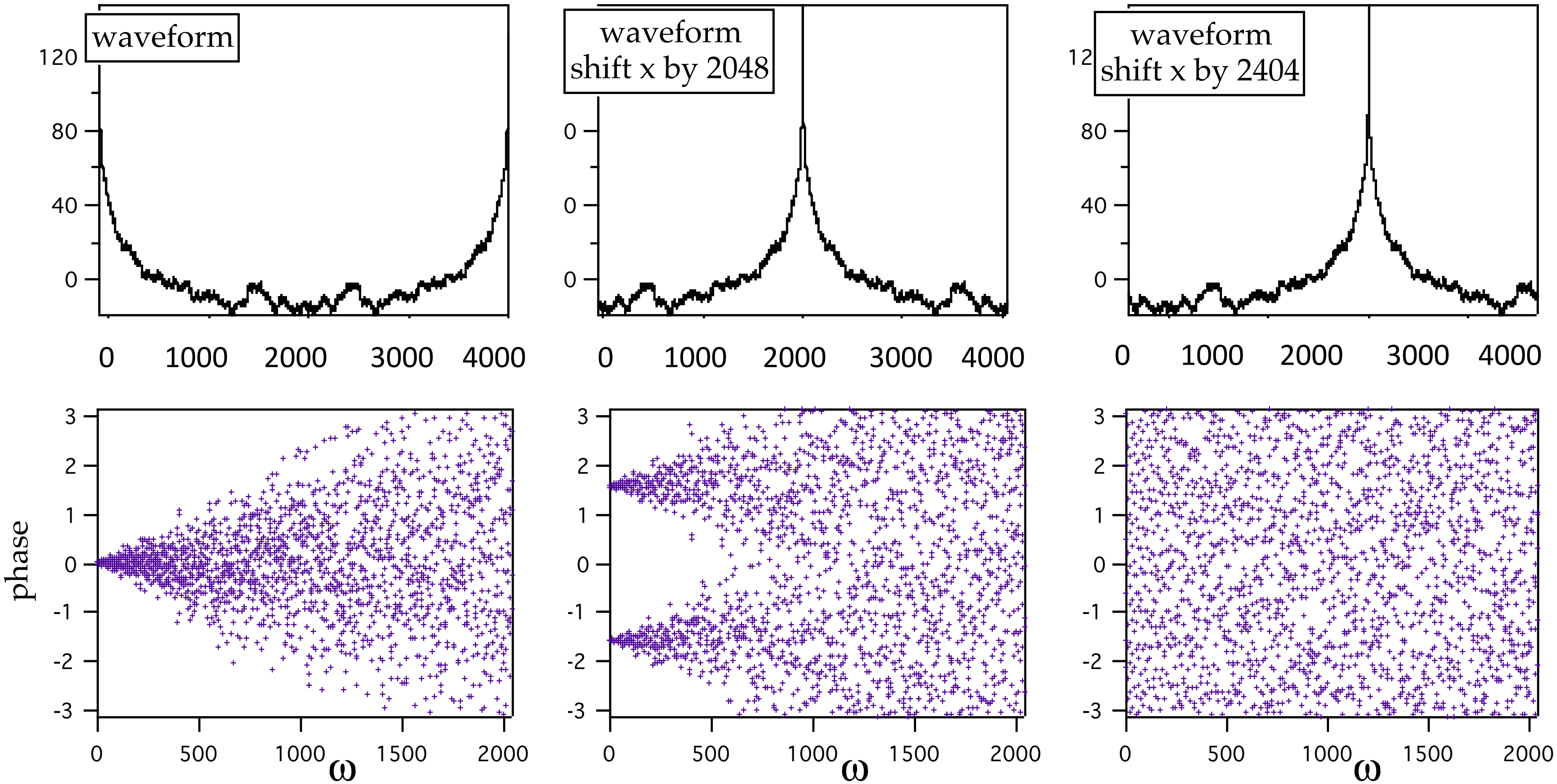
Earth's foreshock: typical parameters

Magnetic field ~ 5 nT
Plasma density ~ 5 /cc
Solar wind velocity ~ 400 km/s
Alfven velocity ~ 50 km/s
Ion gyro frequency ~ 0.1 /s
Plasma frequency ~ 3000 /s

Convection time scale in the foreshock ~ 300 sec

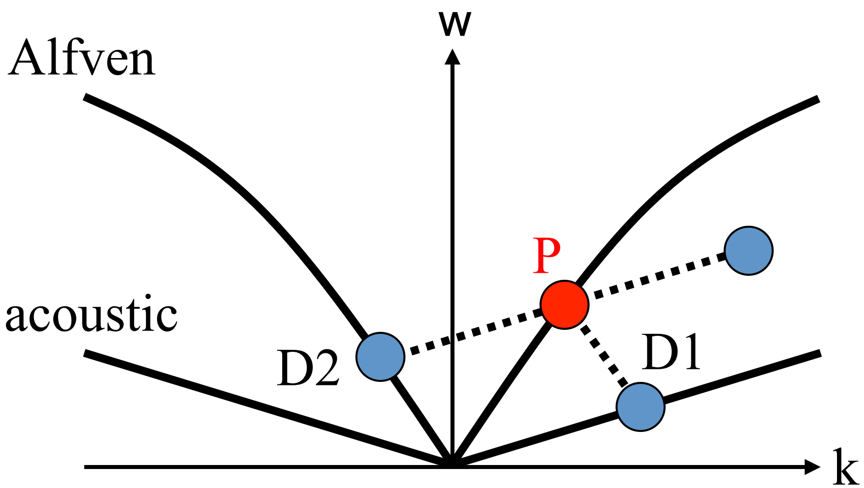
One can make in situ measurement of the sequence:
MHD wave excitation - Nonlinear evolution -
Generation of Turbulence

Phase distribution depends on the coordinates



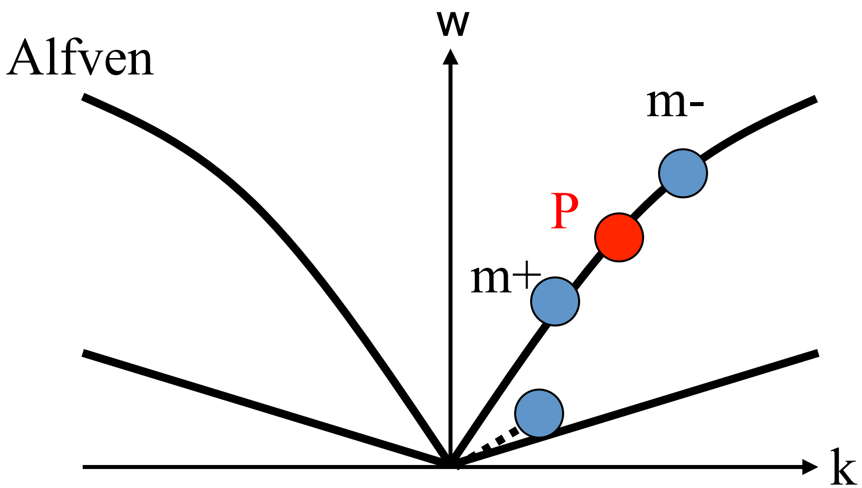
Parametric decay instability :

$$w_p - w_1 = w_2$$
$$k_p - k_1 = k_2$$

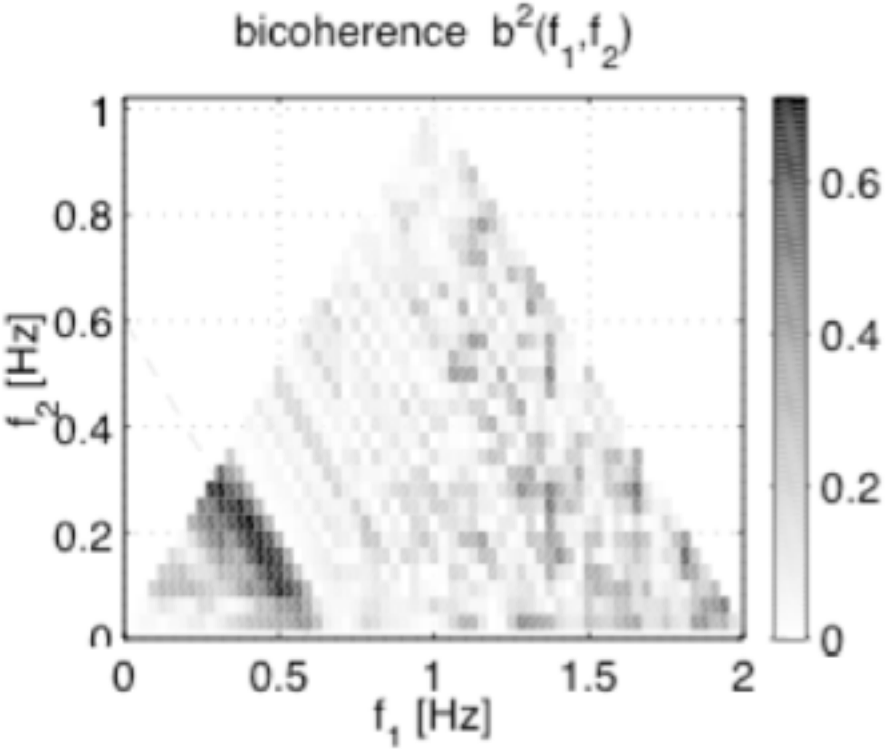


Modulational instability :

$$2w_p = w_1 + w_2$$
$$2k_p = k_1 + k_2$$

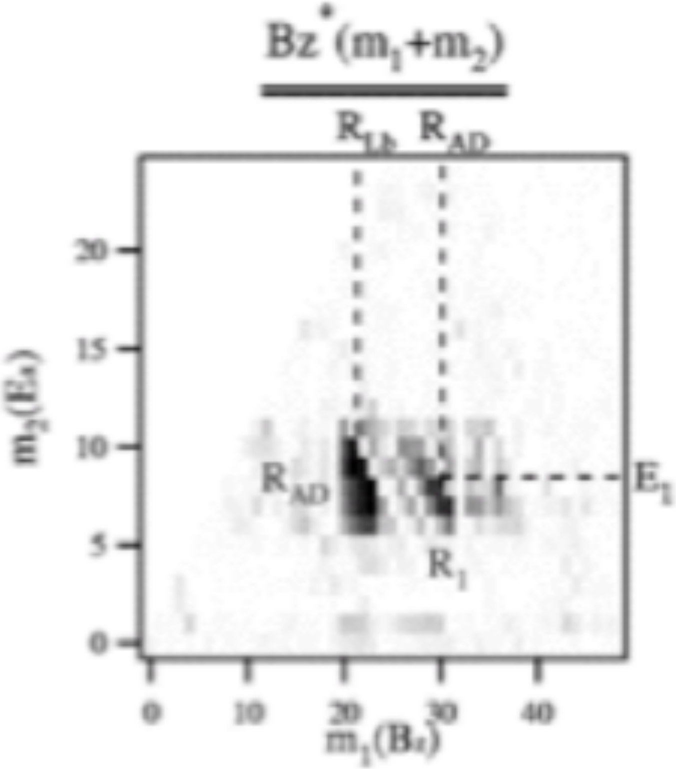


Bi-coherence: examples



Foreshock MHD waves

de Wit et al., 2005



Decay instability of Alfven waves (numerical simulation)

Matsukiyo et al., 2003