#### SWI & MAGDAS SCHOOL

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### Solar Magnetohydrodynamics

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### Dynamic Sun





### Outlook

- Introduction to Magnetohydrodynamics
  - Alfven's theorem (frozen-in)
- Magnetic Structure in Solar Active Region
  - Solar magnetic field
  - MHD equilibrium
  - Magnetic helicity and free energy
  - Force-free field model
- Dynamics in solar coronal magnetic field
  - Solar eruptions (flare and CME)
  - Magnetic reconnection
- What triggers the onset of solar eruptions? [NEW]
  - Ensemble 3D MHD simulation study
  - Hinode Observations
  - Discussion: Predictability of solar eruptions

### Introduction to MHD



**Hannes Alfvén** 

### Plasmas



### **Plasma Dynamics**





$$\hat{F}_{s}(\mathbf{r}, \mathbf{v}, t) = \sum_{j=1}^{\infty} \delta[\mathbf{r} - \mathbf{r}_{j}(t)] \delta[\mathbf{v} - \mathbf{v}_{j}(t)], \quad \hat{K}_{s}(\mathbf{r}, \mathbf{v}, t) = \frac{q_{s}}{m_{s}} [\hat{\mathbf{E}}(\mathbf{r}, t) + \mathbf{v} \times \hat{\mathbf{B}}(\mathbf{r}, t)]$$
where  $\delta[\mathbf{a}] = \delta(a_{x})\delta(a_{y})\delta(a_{z})$ 

$$\hat{\mathbf{J}}_{s}(\mathbf{r},t) = \sum_{s} q_{s} \int \mathbf{v} \hat{F}_{s}(\mathbf{r},\mathbf{v},t) d\mathbf{v}$$
$$\hat{\sigma}_{s}(\mathbf{r},t) = \sum_{s} q_{s} \int \hat{F}_{s}(\mathbf{r},\mathbf{v},t) d\mathbf{v}$$

$$\nabla \times \hat{\mathbf{E}}(\mathbf{r},t) = -\frac{\partial \hat{\mathbf{B}}(\mathbf{r},t)}{\partial t}$$
$$\nabla \times \hat{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c^2} \frac{\partial \hat{\mathbf{E}}(\mathbf{r},t)}{\partial t} + \mu_0 \hat{\mathbf{J}}(\mathbf{r},t)$$
$$\nabla \cdot \hat{\mathbf{B}}(\mathbf{r},t) = 0$$
$$\nabla \cdot \hat{\mathbf{E}}(\mathbf{r},t) = \frac{1}{\varepsilon_0} \hat{\sigma}_e(\mathbf{r},t)$$



 $F_{s}(\mathbf{r}, \mathbf{v}, t) = \langle \hat{F}_{s}(\mathbf{r}, \mathbf{v}, t) \rangle = \int d\{\mathbf{r}_{i}, \mathbf{v}_{i}\} D(\{\mathbf{r}_{i}, \mathbf{v}_{i}\}; t) F_{s}(\mathbf{r}, \mathbf{v}, \{\mathbf{r}_{i}, \mathbf{v}_{i}\})$ ensemble average

$$\mathbf{J}_{s}(\mathbf{r},t) = \sum_{s} q_{s} \int \mathbf{v} F_{s}(\mathbf{r},\mathbf{v},t) d\mathbf{v}$$
$$\sigma_{s}(\mathbf{r},t) = \sum_{s} q_{s} \int F_{s}(\mathbf{r},\mathbf{v},t) d\mathbf{v}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}$$
$$\nabla \times \mathbf{B}(\mathbf{r},t) = \frac{1}{c^2} \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} + \mu_0 \mathbf{J}(\mathbf{r},t)$$
$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0$$
$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = \frac{1}{\varepsilon_0} \sigma_e(\mathbf{r},t)$$

### Macroscopic Variables

the n-th moment in velocity space

$$\mathbf{M}^{n} = \int \mathbf{v}^{n} F_{s}(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$\mathbf{n}_{s}(\mathbf{r}, t) = \mathbf{M}^{0} = \int F_{s}(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$\Gamma_{s}(\mathbf{r}, t) = \mathbf{M}^{1} = \int \mathbf{v} F_{s}(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

$$\mathbf{u}_{s}(\mathbf{r}, t) = \overline{\mathbf{v}}_{s}(\mathbf{r}, t) = \Gamma_{s}(\mathbf{r}, t) / \mathbf{n}_{s}(\mathbf{r}, t)$$

$$\mathbf{c} = \mathbf{v}_{s} - \mathbf{u}_{s} \text{ velocity fluctuation } (速度揺らぎ)$$

$$\widetilde{\mathbf{P}}(\mathbf{r}, t) = \mathbf{m} M^{2} - \mathbf{m} \int \mathbf{c} \mathbf{c} \mathbf{F} d\mathbf{v}$$

$$V$$
  
 $n_i$   
 $n_e$   
 $\chi$ 

 $\mathbf{P}_{s}(\mathbf{r},t) = m_{s}M^{-} = m_{s}\int \mathbf{c}\mathbf{c}F_{s}d\mathbf{v}$  $Q_s(\mathbf{r},t) = \frac{1}{2} m_s \int \mathbf{c} c^2 F_s d\mathbf{v}$ 

### **Macroscopic Equations**

$$\int \mathbf{v}^{n} \left[ \frac{\partial F_{s}}{\partial t} + \mathbf{v} \cdot \frac{\partial F_{s}}{\partial \mathbf{r}} + K_{s} \cdot \frac{\partial F_{s}}{\partial \mathbf{v}} = \left( \frac{\partial F}{\partial t} \right)_{collision} \right] dv$$
  
moment in velocity space  
$$n = 1 \qquad \frac{\partial n_{s}}{\partial t} + \nabla \cdot (n_{s} \mathbf{u}) = 0$$
  
$$n = 2 \qquad \frac{\partial}{\partial t} (\rho_{s} \mathbf{u}_{s}) + \nabla \cdot (\rho_{s} \mathbf{u}_{s} \mathbf{u}_{s}) + \nabla \cdot \widetilde{\mathbf{P}}_{s} - \rho_{cs} \mathbf{a}_{s} = \left( \frac{\partial M_{s}}{\partial t} \right)_{collision}$$
  
$$n = 3 \qquad \frac{\partial}{\partial t} \left( \frac{3}{2} \widetilde{\mathbf{P}}_{s} + \frac{1}{2} \rho_{s} u_{s}^{2} \right) + \nabla \cdot \left\{ \left( \frac{1}{2} \rho u_{s}^{2} + \frac{5}{2} \widetilde{\mathbf{P}}_{s} \right) \mathbf{u}_{s} + Q_{s} \right\} - \rho \mathbf{J} \cdot \mathbf{E}$$
  
$$= \int \frac{m}{2} c^{2} \left( \frac{\partial F_{s}}{\partial t} \right)_{collision} d\mathbf{v}$$
  
s=ion or electron

"closure"

### Single-fluid variables

For the macro-scale variables

 $L >> \lambda_D \qquad \omega << \omega_p \rightarrow n_e \approx n_i \text{ quasi-neutrality 準中性}$ Total mass density

→ X quasi-neutrality 進中性

n,

 $\rho(\mathbf{r},t) = n_e m_e + n_i m_i = n(m_e + m_i) \sim n m_i = \rho_i$ 

Center of mass velocity

$$\mathbf{u}(\mathbf{r},t) = \frac{1}{\rho} (n_e m_e \mathbf{u}_e + n_i m_i \mathbf{u}_i) \sim \mathbf{u}_i$$
  
Electric current density

$$\mathbf{J}(\mathbf{r},t) = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e) \sim -neu_e$$

## Magnetohydrodynamics (MHD) eq.

Cont. eq.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- Eq. Motion  $\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla \cdot \tilde{\mathbf{P}} - \mathbf{J} \times \mathbf{B} = (\frac{\delta M}{\delta t})_{collision}$ • Energy eq.  $\frac{\partial}{\partial t} (\frac{1}{\gamma - 1} \tilde{\mathbf{P}}) \frac{1}{2} \rho u^{2} + \rho U_{grav} + \frac{\varepsilon_{0}}{2} E^{2} + \frac{B^{2}}{2\mu_{0}})$   $+ \nabla \cdot \{\rho \mathbf{u} (\frac{\gamma}{\gamma - 1} \frac{P}{\rho} + \frac{1}{2} u^{2} + U_{grav}) + \frac{\mathbf{E} \times \mathbf{B}}{\mu_{0}} + \mathbf{Q}\} = -\mathbf{J} \cdot \mathbf{E} + \int \frac{m}{2} c^{2} (\frac{\delta F_{s}}{\delta t})_{collision} d\mathbf{v}$
- Generalized Ohm's law

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J}$$
Faraday's law
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

### **Hierarchy of Plasma equations**



### **Scales**

- Gyro-radius  $\rho_s = \frac{v_{th}}{\omega_{s,gyro}}$   $\omega_{s,gyro} = \frac{eB}{m_s}$ • Skin length c  $ne^2$ 
  - $d_s = \frac{c}{\omega_{ps}}$   $\omega_{ps} = \sqrt{\frac{ne^2}{m_s \varepsilon_0}}$
- Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{2ne^2}}$$



### Scales in the solar coronal plasma





 $r_{i} \sim 1 m, \omega_{gi} \sim 10^{6} r_{e} \sim 2x10^{-2} m, \omega_{ge} \sim 10^{9}$ 

 $R_{sun} \sim 7.0 \times 10^8 m$ 

"凍結 (frozen-in)"



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### Magnetic Structure in Solar Active Regions



### Solar Atmosphere

### Multi-layer Structure





#### white light 6x10<sup>3</sup>K



### **Magnetic Structure**



open field

closed field

### Plasma β ratio



$$\beta = \frac{plasma \ pressure}{magnetic \ pressure}$$
$$= \frac{p}{\frac{B^2}{2\mu_0}}$$

### **MHD Equilibrium**



### Linear Force Free Field



• Rectangle system  $\mathbf{B} = B_0(\cos \alpha z, -\sin \alpha z, 0)$   $\nabla \times \mathbf{B} = \alpha B_0(\cos \alpha z, -\sin \alpha z, 0) = \alpha \mathbf{B}$ 

 $\mathbf{B} = B_0(0, J_1(\alpha r), J_0(\alpha r))$ 

Cylindrical system





$$\nabla \times B = (0, -\frac{\partial B_z}{\partial r}, \frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)) = \alpha B_0(0, J_1(\alpha r), J_0(\alpha r))$$



# Helical Structure in the Corona



# Magnetic Helicity (field linkage)



## Helicity is a topological invariant.



Linking Fluxes

 $H = \int \mathbf{A} \cdot \mathbf{B} \, dV$  $= \oint_{C_{i}} \mathbf{A} \cdot d\mathbf{l} \int \mathbf{B} \cdot d\mathbf{S}$  $+\oint_{C_2} \mathbf{A} \cdot d\mathbf{l} \int \mathbf{B} \cdot d\mathbf{S}$  $=2\psi\phi$ 

# Helicity is a topological invariant.



 $H = \int \mathbf{A} \cdot \mathbf{B} \, dV$  $= \oint_{C_{i}} \mathbf{A} \cdot d\mathbf{l} \int \mathbf{B} \cdot d\mathbf{S}$  $+\oint_{C_2} \mathbf{A} \cdot d\mathbf{l} \int \mathbf{B} \cdot d\mathbf{S}$ = ()



Calculate the magnetic helicity of these flux tubes.



- Magnetic helicity is a topological invariant.
- Magnetic helicity is conserved in the ideal MHD systems.

# Taylor's minimum energy state

 When mag. helicity is approximately conserved in the resistive MHD system, the minimum energy state is given by a linear force-free field.

minimize: 
$$E = \int \mathbf{B} \cdot \mathbf{B} dV$$
  
invariance:  $H = \int \mathbf{A} \cdot \mathbf{B} dV$   $\{\nabla \times \mathbf{B} = \alpha \mathbf{B}\}$ 





Taylor's minimum energy state

### **Magnetic Free Energy**



### Force-Free Field in the Corona



### Solar Observatories in Space



# Magnetograph of NOAA10930





#### 2006.12.13 01:42:37





#### 2006\_12\_11 17:00 UT

 

 Total Magnetic Energy Magnetic Free Energy (J)

 1.00E+26

 9.00E+25

 8.00E+25

 7.00E+25

 6.00E+25

 5.00E+25

 4.00E+25

 3.00E+25

1

2

3

4

201 2/9/ 23



33h prior to flare

2006\_12\_11 17:00 UT

Total Magnetic Energy Magnetic Free Energy (J) 1.00E+26 9.00E+25 8.00E+25 7.00E+25 6.00E+25 flare 5.00E+25 4.00E+25 3.00E+25 2.00E+25 1.00E+25 0.00E+00 201 2/9/ 23 1 2 3



# 21h prior to flare

#### 2006\_12\_12 03:50 UT





#### 8h prior to flare

#### 2006\_12\_12 17:40 UT





#### 5h after flare







#### フレア発生 5時間<mark>後</mark>

#### 2006\_12\_13 07:00 UT

Total Magnetic Energy Magnetic Free Energy (J)



### NLFF before & after flare



Schrijver et al. 2008

# Dynamics in solar coronal magnetic field



### **Solar Eruptions**







### **Solar Flares**



2000/06/04 02:05:45



### **Properties of Flares**









0410042926FBin2Bit10p000 50

5万 km



02:52:58 UT

04:45:58 UT

06:00:34 UT

09:06:42 UT



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### **Magnetic Reconnection Model**



# What triggers the onset of solar eruptions?



### Prediction is important.

- For the space weather forecasting
  - When will giant flares and CMEs occur?
  - How is the geo-effectiveness?

- For the promotion of scientific understandings
  - New theories have been established through the efforts for prediction.
    - e.g. Halley's prediction (1682, 1758)





### **Papers for Flare Prediction**

- Poisson statistics (Gallagher et al. 2002, Bloomfield1 et al 2012)
- Bayesian statistics (Wheatland 2005)
- wavelet predictors (Yu et al. 2010a)
- Bayesian networks (Yu et al. 2010b)
- vector machines (Li et al. 2007)
- discriminant analysis (Barnes et al. 2007)
- ordinal logistic regression (Song et al. 2009; Yuan et al. 2010)
- neural networks (Colak & Qahwaji 2009; Yu et al. 2009; Ahmed et al. 2012)
- predictor teams (Huang et al. 2010)
- superposed epoch analysis (Mason & Hoeksema 2010)
- empirical projections (Falconer et al. 2011).

### Skill Score of X-flare prediction

1day	2day	3day	year (events)
-0.068	-0.096	-0.141	2011 (8)
0.112	-0.147	-0.171	2006 (4)
0.242	0.147	0.127	2005 (13)
0.052	-0.001	-0.044	2004 (9)
0.200	0.093	0.076	2003 (17)
-0.037	-0.050	-0.033	2002 (12)
-0.061	-0.034	-0.006	2001 (18)

 $SS = \frac{n_{ff} - (n_q - n_{qq})}{N}$ 

 $n_{f}$ 

Space Weather Prediction Center

http://www.swpc.noaa.gov/forecast\_verification/

#### solar flare (GOES X-ray flux)

### Storage & Release Process





#### stock market (Dou Jones in Black Monday)



http://en.wikipedia.org/wiki/Black\_Monday\_%281987%29

### **Empirical Prediction**



### **McIntosh classification**



Fig. 1. The 3-component McIntosh classification, with examples of each category.

#### McIntosh 1990

### Flare Prob. for each McIntosh class

### Gallacher, Moon, Wang 2002 Sol. Phys.



Figure 4. Derived 24-hour active-region flare probabilities for each of the three McIntosh classification parameters using Poisson statistics.

### Welsch et al. 2009 ApJ

# The total magnetic flux decides the maximum flare size.



### Kusano et al. 2012 submitted to ApJ Ensemble Simulation Study



Box: Rectangle including PIL Initial condition: LFFF BC: imposing the electric field to simulate flux emerging

161 simulations

- 3D MHD (zero-beta model)
  - 256x1024x512 grids (finite difference scheme)
- output: 800 GB/run

### **Simulation Results**



### **Two Structures Triggering Eruption**

 There are the two different structures (OP and RS) favorable to the onset of solar flares.







Zirin and Wang 1993

Green, Kliem & Wallace 2011

# Eruption & Reconnection (OP)

Tether cutting model Moore & Roumeliotis 1992

 $F = r_c I^2 - B_p I$ 

Loss of equirlibrium Forbes & Priest 1995 Torus Instability Kliem & Torok 2006



### **Tether Cutting Scenario**





### Simulation (OP-type)









### The Onset of Storage-and-Release



# Summary

- We find that the <u>two different structures (OP and RS)</u> favorable to the onset of solar flares in terms of the ensemble simulations for different magnetic configurations.
- The detail comparison with Hinode and other observations demonstrates that events indeed occurred, which are well consistent with the two scenarios.
- It means that the solar forecast is possible in terms of the sophisticate magnetic observation. However, the lead time of deterministic forecast of flares must be limited by the time-scale of small magnetic structures and flow driver, which is about several hours, presumably.
- The longer term forecast could be performed in a probabilistic manner.