# Energetics of Magnetic Reconnection

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## Two types of magnetic reconnection

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i) **Spontaneous reconnection** via resistive MHD instability (tearing mode)

ii) **Forced reconnection** under external deformation of an MHD stable magnetic configuration

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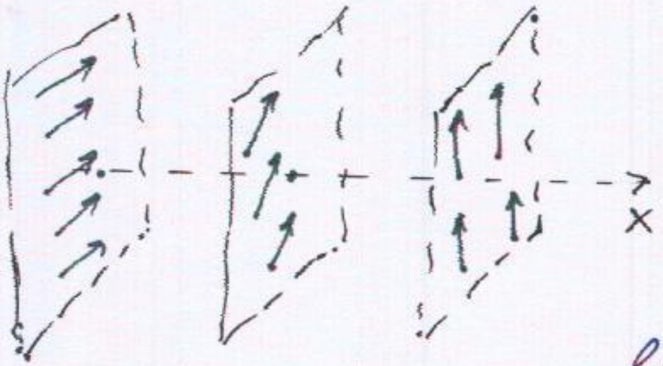
can act as a trigger for **magnetic relaxation**

**Simple example:** sheared force-free magnetic field $\underline{B}^{(0)} = \{0,\ B_0 \sin\alpha x,\ B_0 \cos\alpha x\};\ -a \le x \le +a$

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$$\underline{B}^{(0)} = (\nabla\Psi_0 \times \hat{z}) + B_z^{(0)}\hat{z}$$

$$\Psi_0(x) = \frac{B_0}{\alpha}\cos\alpha x$$

$$B_z^{(0)} = \alpha\Psi_0 \Rightarrow$$

linear force-free field

Tearing perturbation $\rightarrow \Psi(x,y) = \Psi_0(x) + \Psi_1(x)\cos ky$

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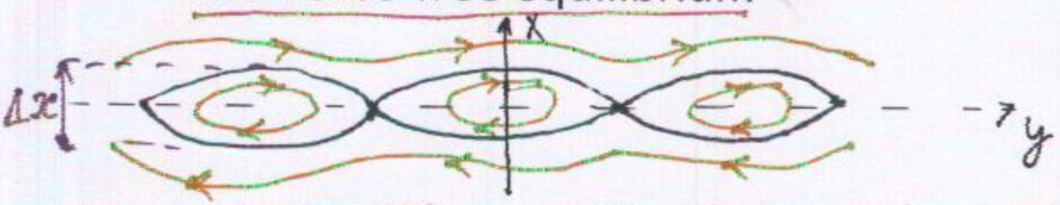
new force-free equilibrium




$\Delta x \sim \sqrt{\Psi_1(0)}$ – width of magnetic islands

$$\Psi_1''(x) + \varkappa^2\Psi_1 = 0; \quad \varkappa^2 = \alpha^2 - k^2 > 0$$

Stability parameter

$$\Delta' = \frac{\Psi_1'(0+\varepsilon) - \Psi_1'(0-\varepsilon)}{\Psi_1(0)} \Rightarrow \Delta' > 0$$

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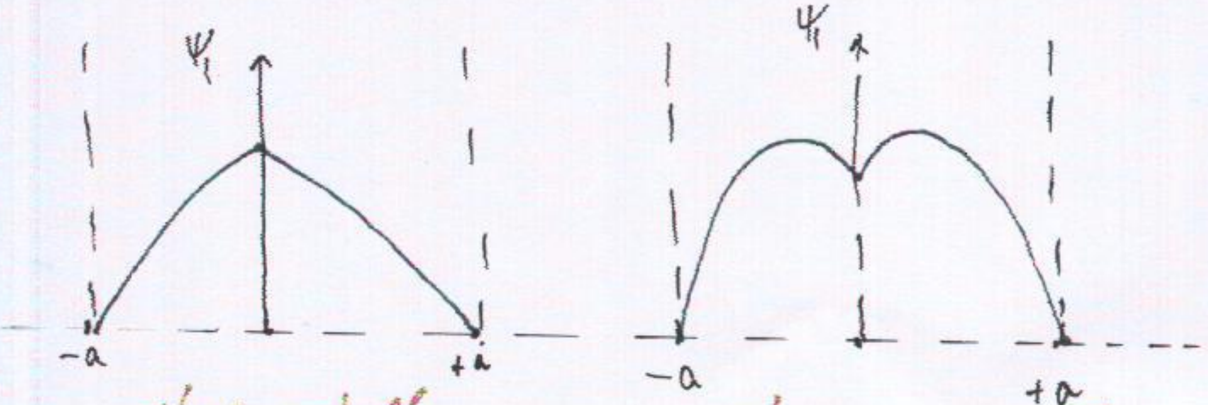
tearing instability




$\Delta' < 0 \Rightarrow$ stable $\qquad$ $\Delta' > 0 \Rightarrow$ unstable

$\Delta' = -2\varkappa \cot(\varkappa a) \Rightarrow$ instability for

$\varkappa a > \frac{\pi}{2} \Rightarrow \alpha_{cr} = \frac{\pi}{2a}$ for modes with $k \to 0$

Physical explanation

Magnetic energy of the system is reduced if $\Delta' > 0$

$$\Delta W_m \propto -\Delta' \psi_1^2(0)$$

Tearing stable configuration: $\alpha < \frac{\pi}{2a} \Rightarrow$

still electric currents are there $\Rightarrow$

possesses some excess magnetic energy

Can it be released by magnetic reconnection?

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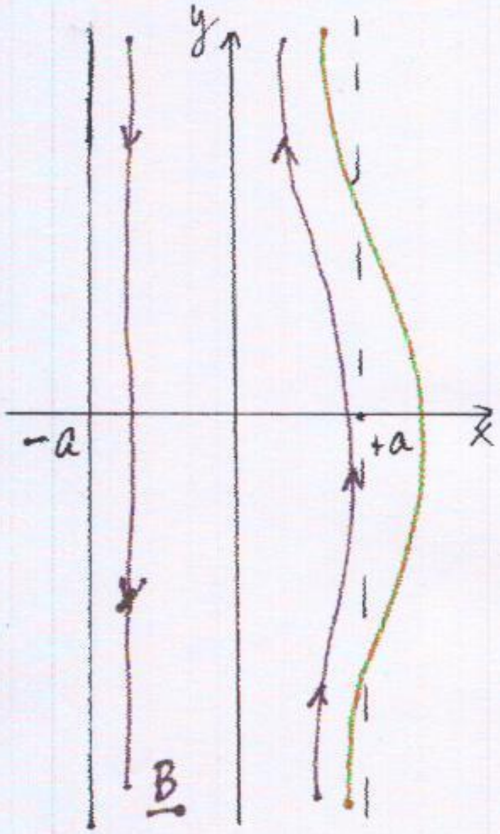
YES, if reconnection is triggered externally!

# Forced magnetic reconnection




Boundary deformation

$$x_b^{(+)} = a + \delta\cos(ky)$$

Slightly ( $\delta \ll a$ )deformed

new force-free magnetic

equilibrium

$$\Psi(x,y) = \frac{B_0}{\alpha}\cos(\alpha x) + \Psi_1(x)\cos(ky)$$

$$\Psi_1'' + \varkappa^2\Psi_1 = 0, \quad \varkappa^2 = \alpha^2 - k^2 > 0$$

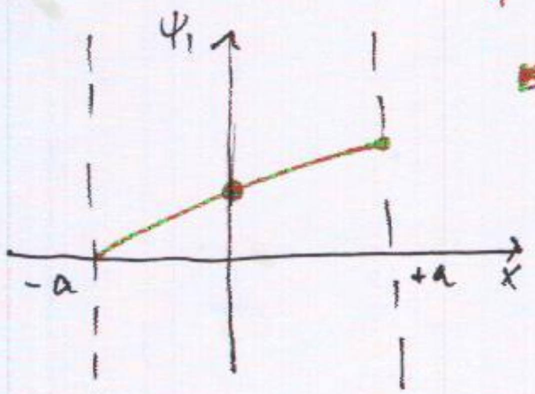
Boundary conditions

$$\Psi(x = x_b) = \Psi_0(a)$$

$$\Psi_1(x=-a) = 0, \quad \Psi_1(x=a) = B_0\,\delta\sin(\alpha a)$$

Regular solution:

$$\Psi_1^{(r)}(x) = \frac{\delta B_0\sin(\alpha a)}{\sin(2\varkappa a)}\sin\left[\varkappa(x+a)\right]$$

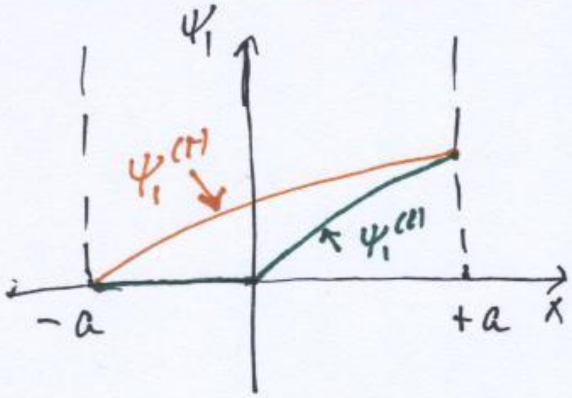



$$\Psi_1(0) \neq 0\,!!$$

magnetic islands present

not allowed in ideal MHD

Ideal MHD solution $\psi_1^{(i)}$ → no reconnection → $\psi_1^{(i)}(0) = 0$



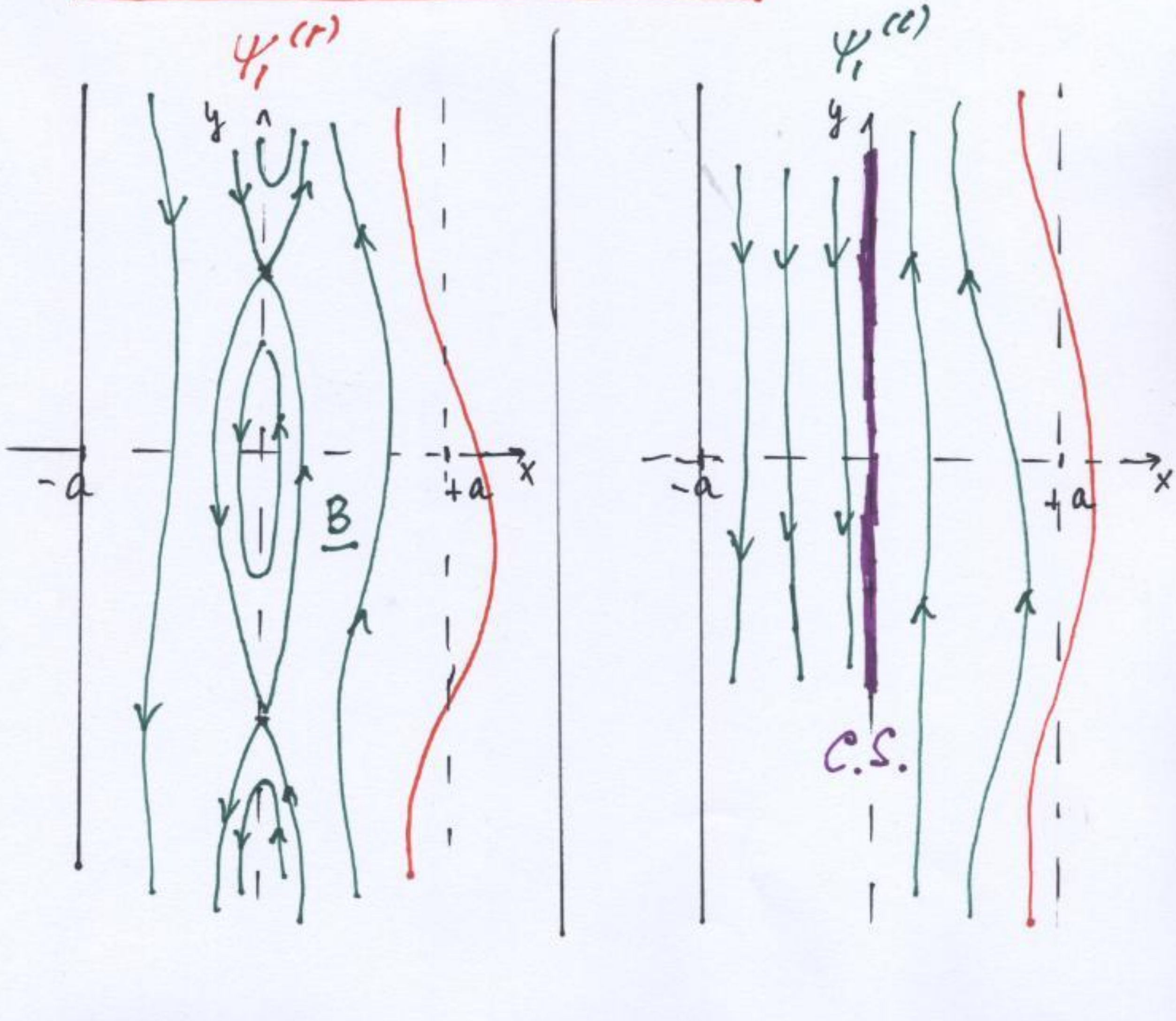

$$\psi_1^{(i)}(x) = B_0 \frac{\delta \sin(\alpha a)}{\sin(\varkappa a)} \sin(\varkappa x)$$

singularity

(discontinuity of $B_y$)

current sheet at $x = 0$.

Two force-free magnetic equilibria



What about their magnetic energy?

Ideal equilibrium: $\psi_1^{(i)}$

$$W_M^{(i)} = W_M^{(0)} + \Delta W_M^{(i)} \rightarrow$$

work of external force required for the boundary deformation

$$\Delta W_M^{(i)} = \frac{B_0^2 \sin^2(\alpha a)}{16\pi a}\delta^2\left[(\varkappa a)\cot(\varkappa a) - (\alpha a)\cot(\alpha a)\right] > 0$$

Reconnected equilibrium: $\psi_1^{(r)}$

$$W_M^{(r)} = W_0 + \Delta W_M^{(r)}$$

$$\Delta W_M^{(r)} = \frac{B_0^2 \sin^2(\alpha a)}{16\pi a}\delta^2\left[(\varkappa a)\cot(2\varkappa a) - (\alpha a)\cot(\alpha a)\right]$$

!! Important point: reconnected state always has a lower magnetic energy

$$\Delta W_M = W_M^{(i)} - W_M^{(r)} = \frac{B_0^2 \sin^2(\alpha a)}{16\pi a}\delta^2(\varkappa a)\left[\cot(\varkappa a) - \cot(2\varkappa a)\right] > 0$$

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Forced magnetic reconnection

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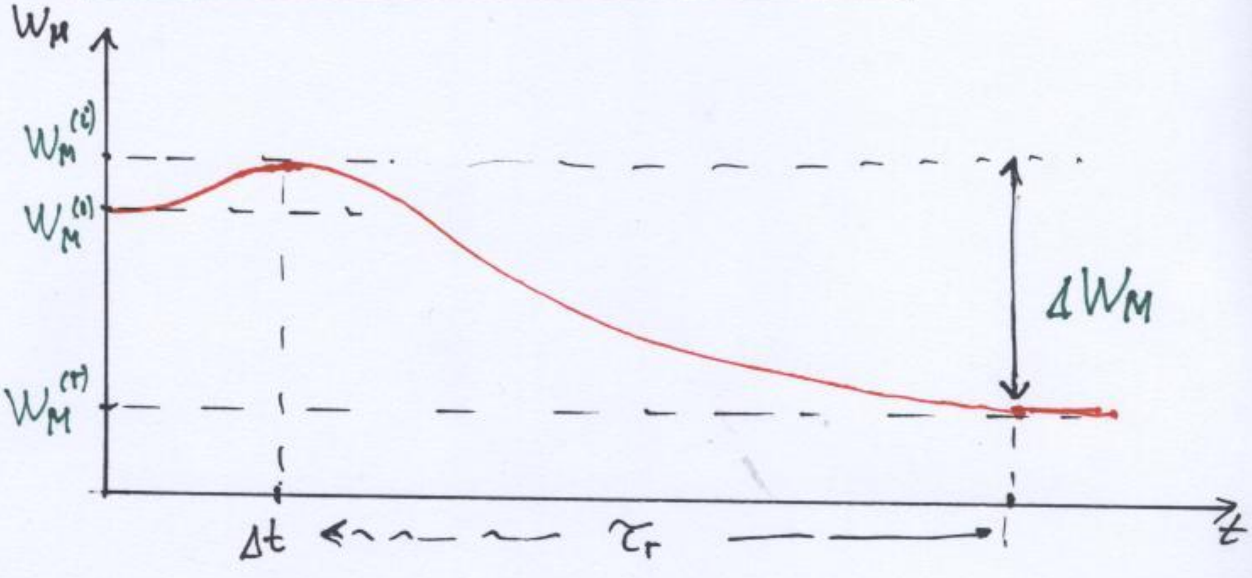
reconnective transition from $\psi_1^{(i)}$ to $\psi_1^{(r)}$

Quasistatic boundary deformation

$$\Delta t \gg \tau_A \sim a/V_A$$

Temporal evolution of magnetic energy




$\Delta t \ll \tau_r \rightarrow$ Ideal MHD evolution

$\Psi_1^{(i)}$ equilibrium is formed

$t \sim \tau_r \sim \tau_A S^{2/3} \rightarrow$ transition to $\Psi_1^{(r)}$ with lower magnetic energy $\rightarrow$

**Reconnective magnetic relaxation**

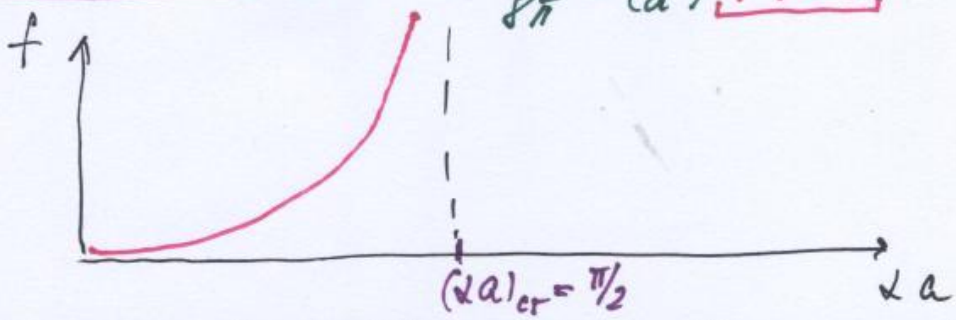
!! **Important point:** released magnetic energy $\Delta W_M$

can greatly exceed $\Delta W_M^{(e)}$

⇓

External perturbation acts as a trigger for internal magnetic relaxation →

released energy $\Delta W_M$ is tapped from excess magnetic energy stored in the initial magnetic configuration: $\Delta W^{(M)} = \frac{B_0^2}{8\pi} a \left(\frac{\delta}{a}\right)^2 f(\alpha a)$,



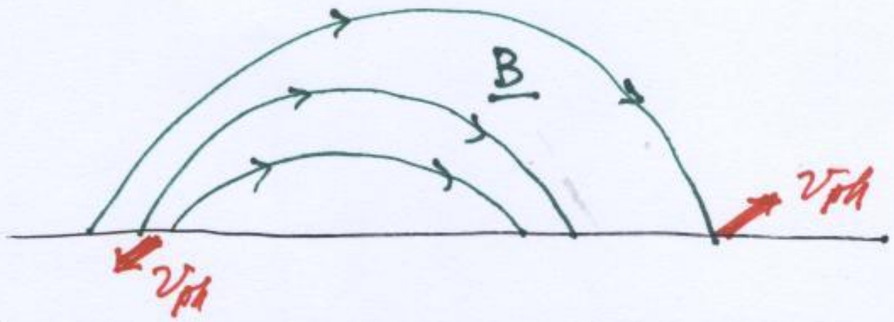

The energy effect of forced magnetic reconnection is strongly amplified for the marginally stable magnetic field →

Link between forced and spontaneous magnetic reconnection

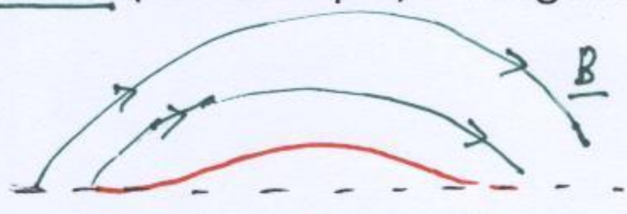
Possible implications for solar corona

- How magnetic energy can be accumulated in the corona without being quickly released by magnetic reconnection?

  Magnetic energy is stored due to one kind of deformation ( for example, by shearing of field lines)




  Then, at some moment, another type of deformation occurs (for example, emergence of new flux)




Forced magnetic reconnection → initially stored magnetic energy is quickly released