(1)

energy loss rate $4\pi R^2 \cdot I = 4 \times 3.14 \times (1.50 \times 10^{11})^2 \times 1.37 \times 10^3$ W

$= 3.87 \times 10^{26}$ W

mass loss rate

$$\dot{m} = \frac{1}{c^2}\dot{E} = \frac{3.87 \times 10^{26}\ \mathrm{W}}{(3.00 \times 10^8\ \mathrm{m/s})^2} = 4.30 \times 10^9 \ \mathrm{kg/s}$$

mass loss rate by the solar wind

$$\rho = m_p n = 1.67 \times 10^{-27} \times 7 \times 10^6 = 1.17 \times 10^{-20} \ \mathrm{kg/m^3}$$

$$4\pi R^2 \cdot \rho u = 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 1.17 \times 10^{-20} \times 4 \times 10^5$$

$$= 1.32 \times 10^9 \ \mathrm{kg/s}$$

total mass loss rate

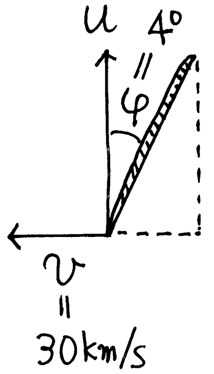
$$(4.30 + 1.32) \times 10^9 = 5.62 \times 10^9 \ \mathrm{kg/s}$$

time to lose all mass

$$\frac{1.99 \times 10^{30}\ \mathrm{kg}}{5.62 \times 10^9\ \mathrm{kg/s}} = 3.54 \times 10^{20}\ \mathrm{s} = \frac{3.54 \times 10^{20}}{86400 \times 365.25}\ \mathrm{y}$$

$$= 1.12 \times 10^{13}\ \mathrm{y} \sim 10 \text{ trillion years}$$

(2)



$$u = v \cot \varphi = 30 \cot 4° = 429 \ \mathrm{km/s}$$

(3)

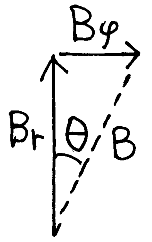
1.
$$\begin{cases} \varphi(r) = \varphi_0 - \dfrac{\Omega}{u}(r - R_s) \\ r = R_s - \dfrac{u}{\Omega}(\varphi - \varphi_0) \approx -\dfrac{u}{\Omega}(\varphi - \varphi_0) \quad \text{for } r \gg R_s \end{cases}$$

Substitute $\varphi = \varphi_0 - 2\pi$ in r

$$r = R_s + \frac{u}{\Omega}\cdot 2\pi \approx \frac{u}{\Omega}\cdot 2\pi$$

$$= \frac{4\times10^5 \times 2\times3.14}{2.80\times10^{-6}} = 8.97\times10^{11}\ \mathrm{m} = \frac{8.97\times10^{11}}{1.50\times10^{11}}\ \mathrm{AU} = 5.98\ \mathrm{AU}$$

2.



$$\tan\theta = \frac{|B_\varphi|}{B_r} = \frac{\Omega}{u} r$$

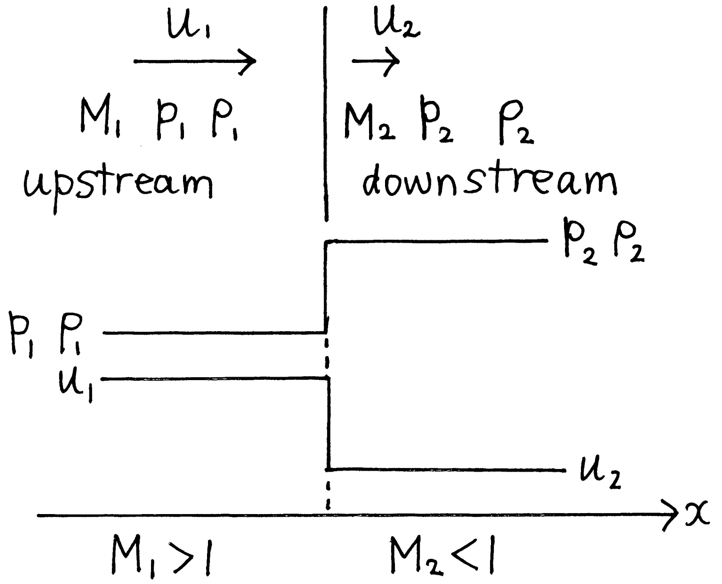
$$= \frac{2.80\times10^{-6}\times8.97\times10^{11}}{4\times10^5} = 6.279$$

$$\theta = 81.0^\circ$$

3.
$$B = B_s\left(\frac{R_s}{r}\right)^2 \sec\theta = 10^{-6}\left(\frac{10\times6.96\times10^8}{8.97\times10^{11}}\right)^2 \sec 81.0^\circ\ \mathrm{T}$$
$$= 3.85\times10^{-10}\ \mathrm{T} = 0.385\ \mathrm{nT}$$

4. $\dfrac{100}{5.98} = 16.7$ times

(4) Rankine-Hugoniot relations



$$\frac{u_2}{u_1} = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_1^2} \qquad ①$$

$$Z \equiv \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1+\frac{2}{M_1^2}} \qquad ②$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \qquad ③$$

$\gamma = \frac{5}{3}$ specific heat ratio (monoatomic gas)

Sonic Mach number $M^2 \equiv \dfrac{\rho u^2}{\gamma p}$

$$Z = \frac{\rho_2}{\rho_1} = \frac{u_1'}{u_2'} = \frac{u_1 - u_s}{u_2 - u_s}$$

From the figure

$Z = 3$, $u_2 = 460$ km/s, $u_1 = 270$ km/s

$$u_s = \frac{Z u_2 - u_1}{Z-1} = \frac{3 \times 460 - 270}{3-1} = 555 \text{ km/s}$$

$$\begin{cases} u_1' = u_1 - u_s = 270 - 555 = -285 \text{ km/s} \\ u_2' = u_2 - u_s = 460 - 555 = -95 \text{ km/s} \end{cases}$$

$\dfrac{u_1'}{u_2'} = \dfrac{285}{95} = 3$ consistent with $Z = \dfrac{\rho_2}{\rho_1} = 3$

From ②

$$M_1^2 = \frac{2Z}{(\gamma+1) - Z(\gamma-1)} = \frac{2 \times 3}{(\frac{5}{3}+1) - 3(\frac{5}{3}-1)} = 9 \qquad \therefore M_1 = 3$$

$$\frac{M_2^2}{M_1^2} = \underset{②}{\left(\frac{\rho_2}{\rho_1}\right)} \underset{③}{\left(\frac{p_1}{p_2}\right)} \underset{②}{\left(\frac{u_2}{u_1}\right)^2} = Z \cdot \frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \cdot \frac{1}{Z^2}$$

$$M_2^2 = \frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \frac{M_1^2}{Z}$$

$$= \frac{\frac{5}{3}+1}{2\times\frac{5}{3}\times 9 - (\frac{5}{3}-1)} \cdot \frac{9}{3} = \frac{3}{11} \qquad \therefore M_2 = \sqrt{\frac{3}{11}} = 0.522$$

From ③

$$\frac{P_2}{P_1} = \frac{2\times\frac{5}{3}\times 9 - (\frac{5}{3}-1)}{\frac{5}{3}+1} = 11$$

(5) The number of times the Sun rotates in a year is

$$\frac{365.256}{25.4}.$$

During this period, the Earth revolves around the Sun once.

The rotational velocity of the Sun as observed from the Earth is

$$\frac{360^\circ \times (\frac{365.256}{25.4} - 1)}{365.256 \text{ days}} = 13.2 \ ^\circ/\text{day}$$

$u = 750$ km/s case

$$13.2 \times \frac{1.50\times 10^8 \text{ km}}{750 \text{ km/s} \times 86400 \text{ s}} = 30.6^\circ$$

$u = 300$ Km/s case

$$13.2 \times \frac{1.50\times 10^8 \text{ km}}{300 \text{ km/s} \times 86400 \text{ s}} = 76.4^\circ$$

(6) X rays

$$\text{time} = \frac{1.50\times10^{11}}{3.00\times10^{8}}\,\text{s} = 500\ \text{s} = 8.33\ \text{min}$$

protons

$$10\ \text{MeV} = 10\times10^{6}\times1.60\times10^{-19}\ \text{J} = 1.60\times10^{-12}\ \text{J}$$

$$\text{velocity} = \sqrt{\frac{2E}{m}} \quad \text{(non-relativistic)}$$

$$= \sqrt{\frac{2\times1.60\times10^{-12}}{1.67\times10^{-27}}}\ \text{m/s} = 4.38\times10^{7}\ \text{m/s}$$

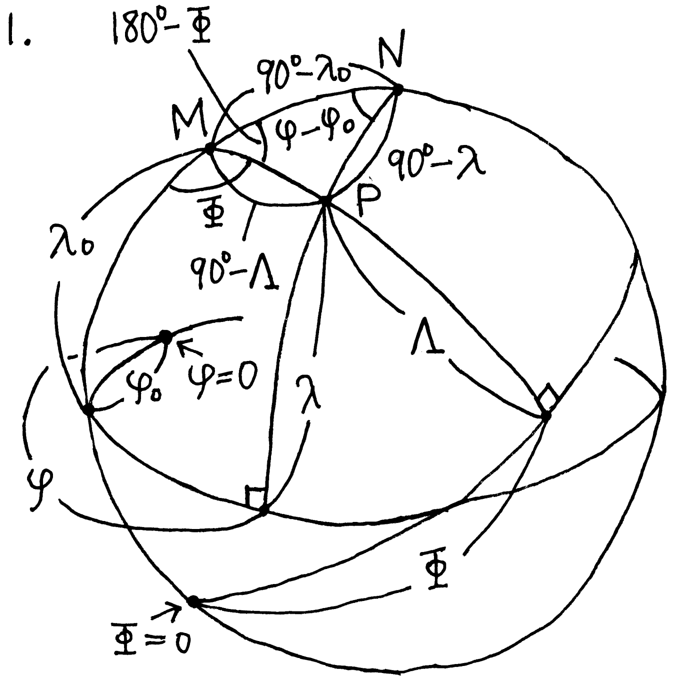
$$\text{time} = \frac{1.50\times10^{11}}{4.38\times10^{7}}\,\text{s} = 3.42\times10^{3}\ \text{s} = 57.0\ \text{min}$$

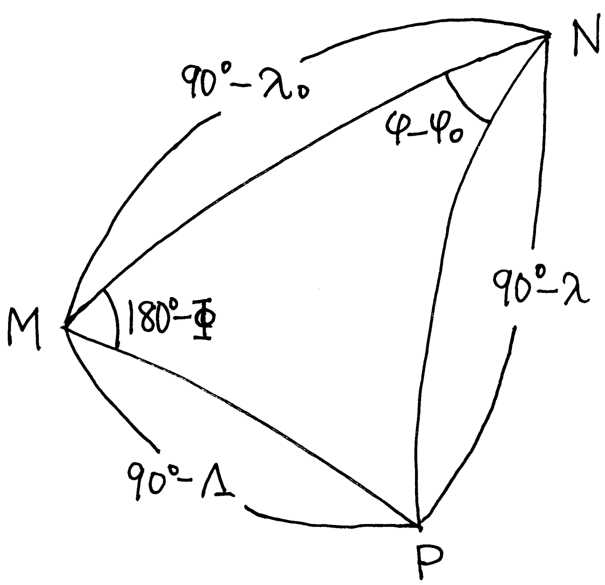
solar wind

$$\text{time} = \frac{1.50\times10^{8}\ \text{km}}{450\ \text{km/s}} = 3.33\times10^{5}\ \text{s} = 3.85\ \text{days}$$

(7)

1.





cosine law

$$\cos(90° - \Lambda) = \cos(90° - \lambda)\cos(90° - \lambda_0) + \sin(90° - \lambda)\sin(90° - \lambda_0)\cos(\varphi - \varphi_0)$$

$$\therefore \sin\Lambda = \sin\lambda\sin\lambda_0 + \cos\lambda\cos\lambda_0\cos(\varphi - \varphi_0)$$

sine law

$$\frac{\sin(90° - \Lambda)}{\sin(\varphi - \varphi_0)} = \frac{\sin(90° - \lambda)}{\sin(180° - \Phi)}$$

$$\therefore \sin\Phi = \frac{\cos\lambda}{\cos\Lambda}\sin(\varphi - \varphi_0)$$

2. Bandung $\lambda = -6.9°$ $\varphi = 107.6°$

North geomagnetic pole $\lambda_0 = 80.0°$ $\varphi_0 = -72.2°$

$$\sin\Lambda = -\sin 6.9° \sin 80.0° + \cos 6.9° \cos 80.0° \cos(107.6° + 72.2°)$$

$$= -0.2907 \qquad \Lambda = -16.9°$$

(8)

1. $B = \sqrt{B_r^2 + B_\lambda^2} = B_0\left(\frac{a}{r}\right)^3\sqrt{1 + 3\sin^2\lambda}$

2. $\frac{dr}{r\,d\lambda} = \frac{B_r}{B_\lambda} = -\frac{2\sin\lambda}{\cos\lambda}$

Let $y = \cos\lambda$. Then $\frac{dr}{r} = -\frac{2\sin\lambda\, d\lambda}{\cos\lambda} = \frac{2dy}{y}$

$\therefore r = c y^2$ $c = \text{const.}$

$r = La$ at $\lambda = 0°$ $\therefore c = La$

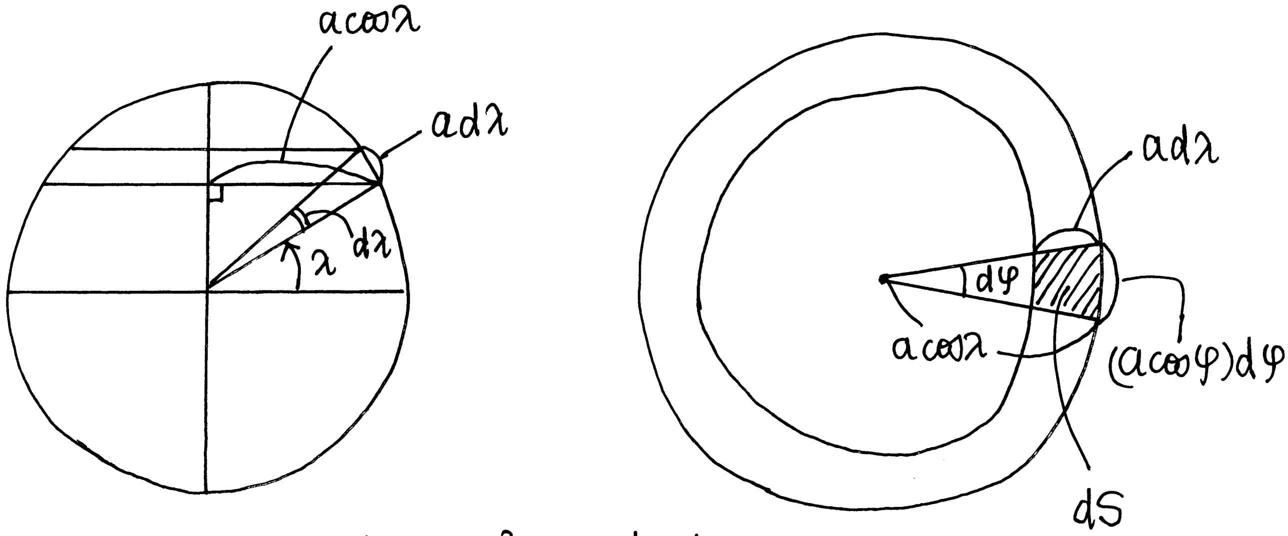
$$\therefore \frac{r}{a} = L\cos^2\lambda$$

3. $\cos\Lambda = \frac{1}{\sqrt{L}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $\therefore\ \Lambda = 60^\circ$

4. Bandung $\Lambda = -16.9^\circ$

$$L = \frac{1}{\cos^2\Lambda} = \frac{1}{\cos^2 16.9^\circ} = 1.09$$

(9)



$$dS = (a\cos\lambda)d\varphi \cdot a\,d\lambda = a^2\cos\lambda\, d\varphi\, d\lambda$$

$$\Phi_{pc} = \iint |B_r|\,dS = a^2\int_0^{2\pi} d\varphi \int_{\Lambda}^{\frac{\pi}{2}} d\lambda \cos\lambda\,|B_r|$$

$$= 2\pi a^2 \int_{\Lambda}^{\frac{\pi}{2}} d\lambda \cos\lambda\,|B_r| = 2\pi a^2 \int_{\Lambda}^{\frac{\pi}{2}} 2B_0 \sin\lambda\cos\lambda\, d\lambda$$

$$= 2\pi a^2 B_0 \cos^2\Lambda$$

$$\frac{\Phi_{pc}(\Lambda = 70^\circ)}{\Phi_{pc}(\Lambda = 75^\circ)} = \left(\frac{\cos 70^\circ}{\cos 75^\circ}\right)^2 = 1.74$$

The magnetic flux increases by a factor of 1.74.

(10) 1. Let $\theta \equiv 90° - \lambda$.

$$W_{dip} = \int_{r \geq a} \frac{B^2}{2\mu_0} dV = \int_{r \geq a} \frac{B_0^2}{2\mu_0} \left(\frac{a}{r}\right)^6 (1+3\cos^2\theta) r^2 \sin\theta \, dr d\theta d\varphi$$

$$= \frac{B_0^2 a^6}{2\mu_0} \int_a^\infty dr \frac{1}{r^4} \int_0^\pi d\theta (1+3\cos^2\theta)\sin\theta \int_0^{2\pi} d\varphi$$

$$= \frac{B_0^2 a^6}{2\mu_0} \cdot \frac{1}{3a^3} \cdot 4 \cdot 2\pi = \frac{4\pi a^3 B_0^2}{3\mu_0}$$

$$= \frac{4\pi \times (6.38\times 10^6)^3 \times (3.00\times 10^{-5})^2}{3\times 4\pi \times 10^{-7}} \text{ J}$$

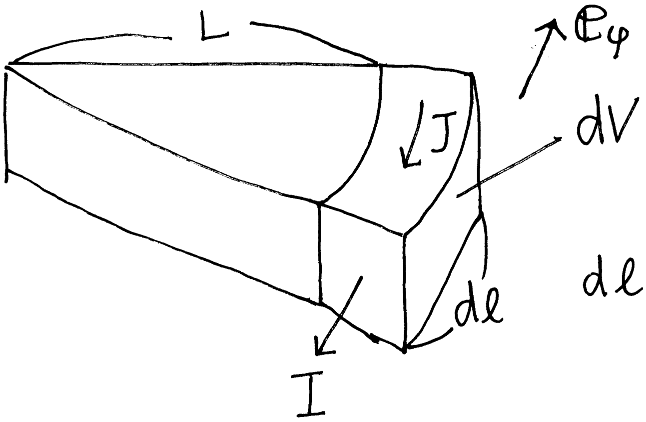
$$= 7.79 \times 10^{17} \text{ J}$$

2. current density J caused by ring current particles with a particular energy W and density n circulating on a given L-shell

$$J(L) = q n v_B = \frac{3L^2 n w}{B_0 a} \quad (-\mathbb{e}_\varphi \text{ direction})$$

total current I

$$I(L) = J \frac{dV}{dl} \qquad \therefore \qquad I dl = J dV$$



dl: circumference element

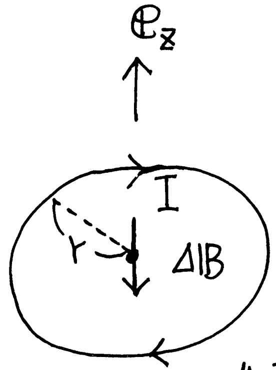
total energy W of all ring current particles at that L

$$W(L) = \int n w \, dV$$

$$\int I\,dl = \int J\,dV$$

$$I \cdot 2\pi La = \int \frac{3L^2 n w}{B_0 a} dV = \frac{3L^2}{B_0 a} W$$

$$\therefore\ I = \frac{3LW}{2\pi B_0 a^2}$$



$$(\Delta \mathbb{B})_z = -\frac{\mu_0 I}{2r}$$

$\Delta \mathbb{B}$ at the Earth's center caused by I

$$(\Delta \mathbb{B})_z = -\frac{\mu_0 I}{2La} = -\frac{\mu_0}{4\pi}\frac{3W}{B_0 a^3}$$

Magnetic disturbance does not explicitly depend on L.

We can replace $W(L)$ by the total energy of ring current particles W_{RC}.

$$(\Delta \mathbb{B})_z = -\frac{\mu_0}{4\pi}\frac{3W_{RC}}{B_0 a^3}$$

Each gyrating particle has a magnetic moment $\mu = \frac{w}{B}$.

$\Delta \mathbb{B}$ at the Earth's center caused by μ

$$(\Delta \mathbb{B})_z = \frac{\mu_0}{4\pi}\frac{\mu}{L^3 a^3}$$

[cf. dipole field $\mathbb{B} = \frac{\mu_0}{4\pi}\frac{M}{r^3}(-2\sin\lambda\, \mathbb{e}_r + \cos\lambda\, \mathbb{e}_\lambda)$]

In the equatorial plane, $B = \frac{B_0}{L^3}$; hence $\mu = \frac{w}{B} = \frac{wL^3}{B_0}$.

$$\therefore (\Delta \mathbb{B})_z = \frac{\mu_0}{4\pi}\frac{1}{L^3 a^3}\frac{wL^3}{B_0} = \frac{\mu_0}{4\pi}\frac{w}{B_0 a^3}$$

For all particles at L,

$$(\Delta \mathbb{B})_z = \int \frac{\mu_0}{4\pi}\frac{n w}{B_0 a^3} dV = \frac{\mu_0}{4\pi}\frac{W(L)}{B_0 a^3}$$

Magnetic disturbance does not explicitly depends on L.

We can replace $W(L)$ by the total energy of ring current particles W_{RC}.

$$(\Delta \mathbb{B})_z = \frac{\mu_0}{4\pi} \frac{W_{RC}}{B_0 a^3}$$

total magnetic field disturbace at the Earth's center

$$(\Delta \mathbb{B})_z = (-3+1) \cdot \frac{\mu_0}{4\pi} \frac{W_{RC}}{B_0 a^3} = -\frac{\mu_0}{2\pi} \frac{W_{RC}}{B_0 a^3}$$

$$= -\frac{2}{3} \frac{W_{RC}}{W_{dip}}$$