

# Solar Cosmic Ray Acceleration and Propagation

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**Abstract.** The GOES data for emission of flare protons with the energies of 10 - 100 MeV are analyzed. Proton fluxes of  $\sim 10^{32}$  accelerated particles take place at the current sheet decay. Proton acceleration in a flare occurs along a singular line of the current sheet by the Lorentz electric field, as in the pinch gas discharge. The duration of proton flux measured on the Earth orbit is by 2 - 3 orders of magnitude longer than the duration of flares. The high energy proton flux from the flares that appear on the western part of the solar disk arrives to Earth with the time of flight. These particles propagate along magnetic lines of the Archimedes spiral connecting the flare with the Earth. Protons from the flare on the eastern part of the solar disk begin to register with a delay of several hours. Such particles cannot get on the magnetic field line connecting the flare with the Earth. These protons reach the Earth, moving across the interplanetary magnetic field. The particles captured by the magnetic field in the solar wind are transported with solar wind and due to diffusion across the magnetic field. The patterns of solar cosmic rays generation demonstrated in this paper are not always observed in the small ( $\Phi \leq 1 \text{ cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}$ ) proton events.

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## Introduction

The solar flare is a very complicated phenomenon (Podgorny and Podgorny, 2012a; 2012b) which consists of soft thermal X-ray emission in the corona, hard X-ray emission produced by the fast electron beams, coronal mass ejections, relativistic proton emission and electromagnetic radiation in the wide spectral region. Flares are very individual and some of effects are not presented in each flare. Only about 20% of large flares (class X and even less class M) are accompanied by the flux of relativistic protons. Duration of proton events recorded on the Earth's orbit is typically 2 - 3 days. A part of fast protons precipitates on the Sun and produces nuclear reactions with  $\gamma$ -ray emission. The duration of  $\gamma$ -ray emission 20 - 30 min coincides with duration of flare X-ray. Despite decades of the effort, the relativistic proton emissions (solar cosmic rays) remain the least studied manifestation of flares. The origin of solar cosmic ray has been investigated for a long time and at least three options have been discussed (1) the acceleration due to the shock in the corona and interplanetary space (Reams 1990, 1999; Tylka and Lee, 2006; Tulupov et al., 2012), (2) the acceleration in the flare process (e.g. Klein and Trottet, 2001), and (3) the flares and shock acceleration (Cane, 2006, 2010).

The main information about physics of solar cosmic rays is obtained from measurements of the accelerated protons flux, which arrive at the Earth's orbit, and from the registration of  $\gamma$ -radiation produced by the nuclear reactions caused by fast protons in the Sun. The world network of neutron monitors can measure spectra in the range of 0.5 - 20 GeV, and spacecrafts GOES measure proton energy in intervals of 10, 50 and 100 MeV. The high energy proton flux reaches the Earth's orbit propagating in the

interplanetary plasma with the frozen-in magnetic field. In the interplanetary space the mean free path estimated for Coulomb collisions of high energy protons exceeds the distance from the Sun to the Earth (AU), and the Larmor radius is significantly less than AU. Such high energy charged particle that captured by the magnetic field line of Archimedes spiral lines connecting the flare and Earth environments is moving along this line and arrives to the Earth after time of flight. These particles produce the prompt component of solar cosmic rays. They possess information about the spectrum of protons accelerated in the flare. The measurements of prompt component proton flux with the world network of neutron monitors show the exponential spectrum  $\exp(-W/W_0)$  (Podgorny et al., 2010a; 2010b). The typical  $W_0$  is  $\sim 0.6$  GeV.

However, the proton beam instability can arise, and the effective the mean free path is decreased (Palmer, 1982; Droege, 2000). Traveling time along the field line is also decreased, and the long tail can be produced by delayed protons, moving along the field line connected the flare and the GOES space craft. The possibility of proton beam instability developed in the space is considered by (Istomin, 2010).

Each proton event is observed after the solar flare. MHD numerical simulation of the magnetic field above the active region that caused the flare has demonstrated formation of a current sheet before the flare. All conditions of numerical simulation were set from the measurements of the active region dynamics. The method of test particles is used to obtain the spectrum of accelerated protons in the current sheet. The calculated spectrum coincides with the measured one at the magnetic reconnection velocity  $\sim 2 \times 10^7$  cm/s (Podgorny et al., 2010b). Thus evidence of the relativistic protons acceleration in the flare current sheet is obtained.

The protons captured by magnetic field lines that do not connect the flare and Earth environment can arrive to the Earth environment moving across the magnetic field. These protons are measured by neutron monitors with delay after traveling with solar wind and due to diffusion across the magnetic lines at particle scattering on the magnetic field inhomogeneities.

These delayed protons demonstrate the power spectrum  $W \sim \gamma^{-5}$ . The spectrum of the protons of the large events measured with the worldwide network of neutron monitors extends up to 20 GeV (Hurford et al., 2003; Balabin et al., 2005; Podgorny et al., 2010a; Podgorny et al., 2010b).

The proton fluxes with not relativistic energy that associated with flares are demonstrated in the GOES experiment in three energetic intervals ( $W > 10$  MeV,  $W > 50$  MeV and  $W > 100$  MeV). In the literature the possibilities are considered of proton acceleration in shock waves and particles interaction with the magnetic turbulence excited by the solar wind. (Kozarev, et al., 2013, Sokolov, et al., 2004; Mewaldt et al., 2012).

About a dozen single large proton events  $\Phi \geq 1 \text{ cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}$  can occur during the solar cycle. However, sometimes one - two times during the solar cycle a series of several GOES large proton events is observed lasting more than 10 days appeared. The proton flux from such series can be comparable with the sum of all the individual fluxes of proton events during a solar cycle. The series proton events registered with GOES in October - November, 2003 is shown in Figure 1. The flares occur in different active regions of the solar disk. This series of proton events begins with the event with the steep front is caused by the western flare X1.2 (N04W43), which has appeared after the eastern flare (S15E43) of the same power. This eastern flare doesn't produce a proton event. Two large flares of November 3 also do not produce a new proton flux. Thus not all large flares are accompanied by proton emission. As follows from the Figure 1 the proton events demonstrate different shapes, different rise times and various flux magnitudes. The superposition of the proton fluxes from various flares in a series of flares makes the analysis of the behavior of individual proton events impossible, so in this paper we investigate the behavior of single proton events.

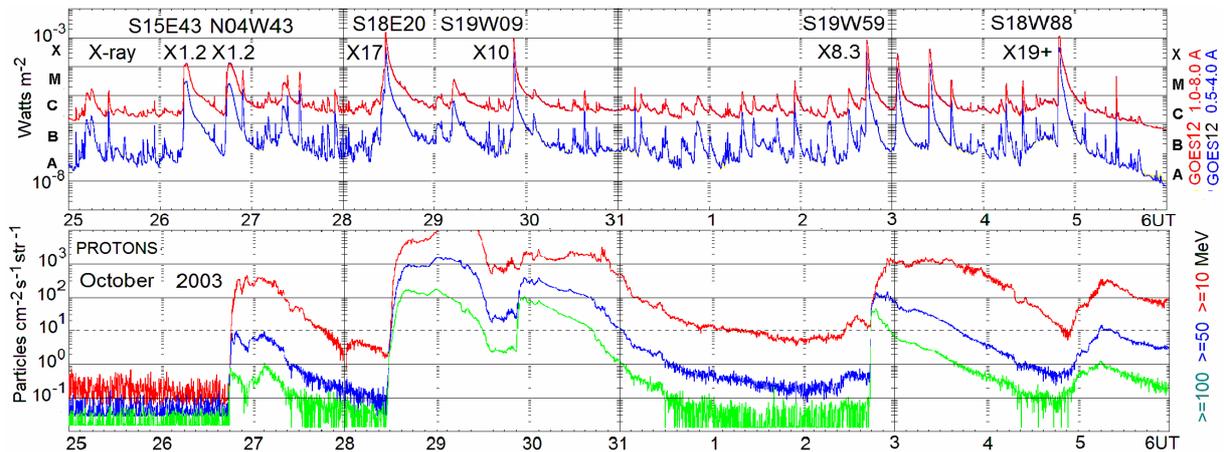


Figure 1. The series of proton flares according to the GOES spacecraft in October - November, 2003.

### Flare protons acceleration

The energy of the charged particles in the interplanetary plasma can be obtained only at particle movement along the electric field  $MdV/dt = eE + e(V \times B)/c$ . The change of the energy of a particle moving in the field is  $dW = eE \cdot dr$ . The electric field may be of different origin: the field of another charged particle (at a collision), space charge field, in particular, due to polarization of charges in the plasma, the field induction  $dB/dt$ , the Lorentz electric field in magnetic clouds, etc. At the scattering by inhomogeneities of the magnetic field the particle can change the energy only due to motion in the electric field associated with magnetic field fluctuations. It was clearly formulated by Berezhko and Krymsky, 1988: "Possibility of charged particles acceleration in the plasma is the related with electric fields." The change of the energy at scattering by a magnetic fluctuation is

determined by the magnitude and the space scale of the electric field in such fluctuations.

Two possibilities of solar cosmic rays generation are discussed in the literature:

a). Proton acceleration along a singular line in the solar flare electric field  $E = -V_{rec} \times B_{cs}$  at fast magnetic reconnection (Podgorny and Podgorny, 2012a; 2012). Here,  $V_{rec}$  is the velocity of magnetic reconnection in a current sheet;  $B_{cs}$  is the magnetic field of this decaying current sheet. The scheme of proton acceleration in a flare current sheet is shown in Figure 2. Such acceleration has been observed in the laboratory experiments on the fast compression of a plasma column by the magnetic field due to rapidly increasing of the current in the electric gas discharge (the pinch effect). Particles acceleration takes place by the Lorentz electric field directed along the axis of the gas discharge (Artsimovich, 1964; Koval'skii et al., 1960).

The maximum energy of the protons accelerated in the current sheet of a solar flare can be estimated as follows. At a typical velocity of magnetic reconnection in a flare  $V_{rec} = 2 \times 10^7$  cm/s, the magnetic field in the current sheet  $B_{cs} \sim 100$  G, and the length  $L$  of the current sheet order of  $10^9$  cm, the maximum energy of protons accelerated in the current sheet can be estimated as  $W = L V_{rec} B_{cs} / c \sim 20$  GeV. This estimation is corresponded to the maximum energy of the solar proton measured at a flare by the neutron monitors and to results of the numerical MHD simulation of proton acceleration in the flare current sheet (Balabin et al. 2005; Podgorny et al., 2010a; 2010b).

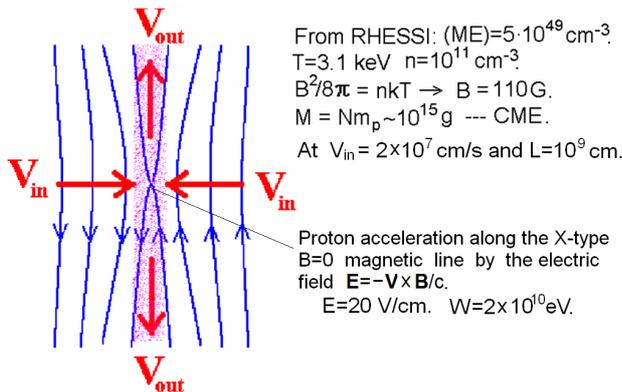


Figure 2. The plasma velocity vectors and magnetic field lines in the current sheet at reconnection. The electric field  $-V \times B / c$  is directed perpendicular to the figure plane.

b). The Fermi acceleration by the Lorentz electric field  $E_{shock} = -V_{shock} \times B_{shock} / c$  in a shock wave, that appeared in the interplanetary space, driven by coronal mass ejection after a flare. Here,  $V_{shock}$  and  $B_{shock}$  are the shock velocity and the magnetic field of the shock wave, driven by coronal mass ejection. It is supposed in several papers (Reams 1990, 1999; Tulupov et al., 2012) that solar cosmic ray protons obtain their energy in the interplanetary space at interaction with shock waves and magnetic clouds. The scheme of not relativistic particle acceleration by a perpendicular shock wave is shown in Figure 3a. Particle that moves towards the magnetic cloud (shock) with the velocity  $V$  is entering in the magnetic cloud. This particle moves along a part of the Larmor circle and gains the energy  $\delta W = 2\rho E_e$  in the Lorentz electric field  $E_{shock}$ . Here,  $\rho$  is the Larmor radius. As a result of reflection from the magnetic cloud the particle velocity increases by  $2V_{shock}$ . If the particle is captured between two moving to each other magnetic clouds, such particle gains the energy at each reflection.

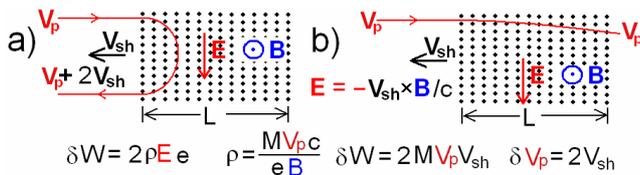


Figure 3. a). The scheme of the Fermi acceleration of the first type. b). Particle reflection is impossible, if the Larmor

radius has the larger size than magnetic cloud, the particle crosses the shock wave.

The maximum energy attainable at acceleration by reflection from the magnetic cloud corresponds to the condition of equality of the Larmor radius to the size of the magnetic plasma cloud. At higher energy the proton reflection from the magnetic field cloud is impossible, and the particle crosses the magnetic cloud. Such proton cannot be accelerated by reflection from the magnetic cloud by the Lorentz field to the large energy (Figure 3b). For a typical shock wave in the interplanetary space near the Earth orbit the magnetic field  $B_{shock} = 10^{-4}$  G and the proton energy maximum  $\sim 20$  GeV, the Larmor radius is  $\rho = R / 300B \sim 10^{12}$  cm, which is several orders of magnitude bigger than the thickness of the observed shock waves. Here  $R(V)$  is the relativistic proton rigidity. Thus, the reflection from the shock wave in the interplanetary space cannot be a source of the observed relativistic protons appearing after a flare. However, this estimate does not deny the possibility of particle acceleration by shock waves in the strong magnetic field near the Sun.

According to Berezhko and Krymsky, 1988 the proton under certain condition can be captured in the strong magnetic field  $B_{shock}$  of a shock wave front and gain energy due to the gradient drift in the shock front, moving along the electric field  $E_{shock} = -V_{shock} B_{shock} / c$ . The conservation of the proton adiabatic invariant in the shock front is supposed, as shown in Figure 4, taken from Berezhko and Krymsky, 1988. To explain the acceleration of protons to high energies these authors consider the possibility of multiple returns of particles in the acceleration region inside the front due to scattering at magnetic fluctuations. However, such mechanism of proton acceleration also requires the Larmor radius to be smaller than the shock front. The effect of this mechanism is possible for a certain interval of energies close to the Sun.

**Large fluxes of solar cosmic rays.**

Figure 1 shows that only a part of the X class flares are accompanied by the great flux  $\Phi \geq 1$  cm<sup>-2</sup> ster<sup>-1</sup> s<sup>-1</sup> of relativistic protons registered at the Earth orbit with the GOES spacecraft. Different possibilities of solar proton acceleration are discussed. It is declared in the paper (Tulupov et al., 2012) that proton acceleration take place in the interplanetary space. Reams (1990, 1999) concludes that "proton prolonged front of time-intensity profile are explained by models of prolonged acceleration in an interplanetary shock". But analysis of long-term measurements of large relativistic proton fluxes ( $\Phi \geq 1$  cm<sup>-2</sup> s<sup>-1</sup> ster<sup>-1</sup>) demonstrate that appearing of different time dependence of the flux front (steep front and prolonged front) strongly depends on the flare position on the solar disk. The typical examples of the single large proton fluxes generated by the western and eastern flares are shown in Figure 5.

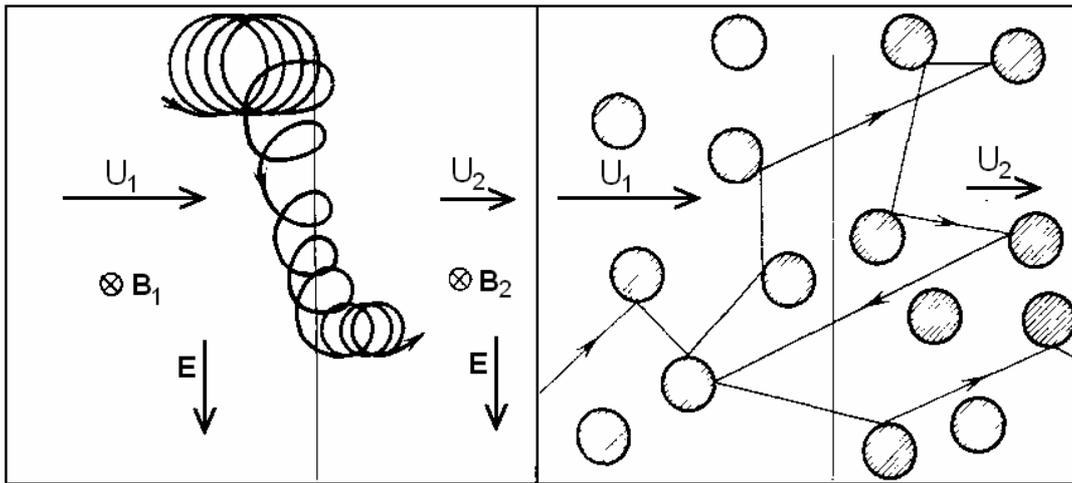


Figure 4. The drift trajectory of a proton in the shock front. The proton returns inside the front due to scattering on the magnetic field inhomogeneities.

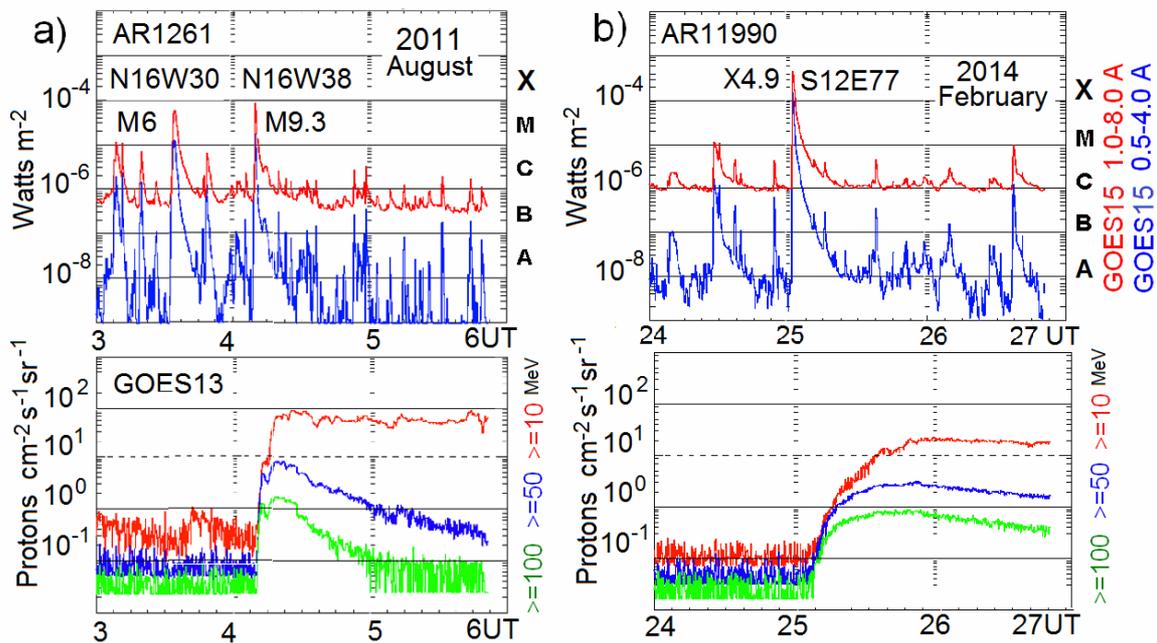


Figure 5. At the top: a) X-ray emission from the western proton flare, b) X-ray emission from the eastern proton flare. Below: the typical large proton fluxes from these flares according to GOES measurements.

The front of the proton pulse, which observed on the western side of the solar disk, appears just after beginning the flare M9.3 N16W38 (Figure 5a). The similar scenario is demonstrated when the first proton flux appeared after the flare X1.2 in western active region N04W43 (Figure 1). The typical proton flux arriving from a flare that occurred in the West of the Sun has a steep front with duration of about 20 min. The proton flux comes from the western flares to the Earth with a delay about 20 min relative to the beginning of the flare. So, the delay is determined by the time of the particle passage from the Sun to the Earth without collisions. The fast protons reach the Earth orbit moving with their velocity, apparently, along the Archimedes magnetic line. The velocity vector of the particle in the flux front is directed, apparently, along a

magnetic field line. These protons correspond to the prompt component of relativistic protons, which according to neutron monitor measurements (Podgorny et al., 2010a; 2010b), demonstrates the exponential spectrum  $\exp(-W/W_0)$ . However, the average duration of the proton pulses is order of 3 days, in spite of the flare duration, determined from soft and hard X-ray radiation measurements, is order of 20 minutes (Figures 1 and 5). The duration order of 20 minutes is also typical for pulses of  $\gamma$ -radiation (Chupp and Ryan, 2009; Kuznetsov, 2011) produced (mainly 2.222 MeV) at nuclear reactions in the Sun initiated by the accelerated protons. The particles registered with the GOES spacecraft that arrived at the end of the proton pulse are not captured by a magnetic field line connecting the flare and the Earth. These particles

propagate in the interplanetary medium due to different mechanism: they are transferred by solar wind velocity, or by diffusion across the interplanetary magnetic field lines. The proton pulse duration is observed in the range 1 – 10 days.

The 3 days duration of the proton pulse is typical for the most of proton fluxes including proton pulses durations in the series of proton fluxes, as it is shown in the Figure 1. The long duration of measured proton flux is shown also in (Malandraki, 2015; Podgorny, Podgorny, 2015). The 3 days duration is equal to the drift time of the protons from the Sun to the Earth with the solar wind velocity order of  $5 \times 10^7$  cm/s. Some particles of the proton flux can arrive earlier, apparently, propagating a part time along a field line, but the other time they drift across the field. The some protons could propagate across the magnetic field faster than the solar wind due to diffusion in the magnetic field at particle scattering by the magnetic inhomogeneities. The delayed relativistic proton components demonstrate the power spectrum  $w^{-\gamma}$ ,  $\gamma \sim 5$  (Podgorny et al., 2010a; 2010b). The change of the spectrum takes place, apparently, due to scattering by the magnetic inhomogeneities.

In the rare cases, the shape of the proton pulse front from a western flare is different than the front of event 2003 November 23 shown in Figure 5a. Sometimes the front of the proton flux is prolonged up to several hours. This happens when the proton flare occurred immediately after large flares and coronal mass ejections. The helical structure of the interplanetary magnetic field in such a case could be distorted. During this unique series of flares 6 large proton events have been observed in the time interval between 27.10.2003 and 06.11.2003 (Figure 1). The interplanetary field has been apparently strongly distorted, and the duration the front of the proton flux

of the giant western flare X19 is increased up to 10 hours.

Unlike western proton flares, the protons from eastern flares and the flares that occurred near the central meridian are beginning to register with the delay relative to the flare in few hours (Figure 5b). The proton flux front from the eastern flare is never steep. Protons from the eastern flare are not able to come to the Earth along an Archimedes spiral line. They can be transported by the solar wind across the magnetic field, and the flux front can be expanded by proton diffusion across the field lines due to scattering with the magnetic fluctuations.

The complexity of eastern flare proton trajectories in the interplanetary medium impedes the proton transfer to the Earth. So the flux of high energy protons from the eastern flares must be smaller than the proton flux along magnetic field lines from the western flares. Drifting across the magnetic field, the protons from the eastern flares are not always able achieve the near-Earth space. The long-term measurements by the GOES device show that the number of registered large proton events from western flares is by order of magnitude greater than the number of proton events from eastern flares. Protons from some eastern flare cannot reach the Earth environment.

### Small fluxes of solar cosmic rays.

The small fluxes ( $\Phi \leq 1 \text{ cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}$ ) of accelerated protons appear on the Earth orbit with approximately the same frequency as the large ones ( $\Phi \geq 1 \text{ cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}$ ). In contrast to the large proton events, the most of small proton events do not reveal a clear relation to any specific flare. Two examples of small proton events and X-ray pulses of the flares are shown in Figure 6.

Pulses of small proton flux usually appeared on the background of a number of small flares, mainly of the C class flares;

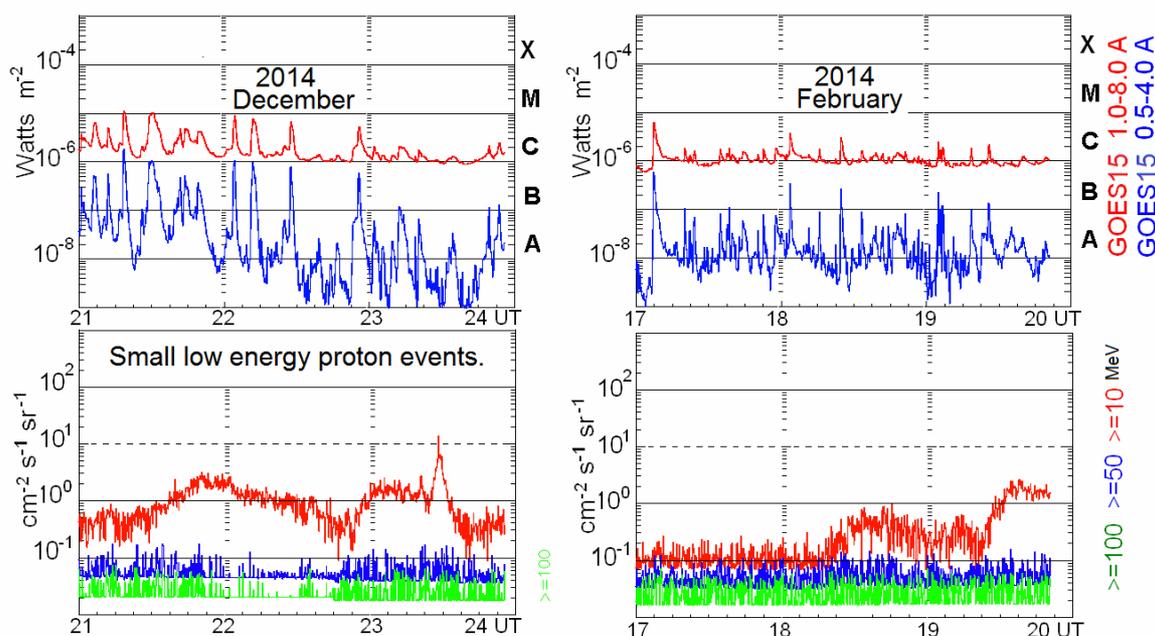


Figure 6. Above - X-ray emission from flares. Below - a typical small flux of protons according to the GOES measurements.

however, any correlations of small proton events with a specific flare are not detected. The duration of observed small proton events, as the large ones, are days, but their structure is purely individual. No spatial characteristics of small events are observed. The lack of good correlation of small proton events with a specific flare observed on the solar disk, apparently, indicates the generation of these events on the back side of the Sun. These protons can arrive along helical magnetic field lines from the back side.

## Conclusions.

Several authors (Ramaty and Morphy, 1987) paid attention to solar cosmic rays generation due to particle acceleration in solar flares or (Reames, 2013) with "resonant stochastic acceleration, related to magnetic reconnection involving open field lines", and also with particle acceleration in shock waves (Ng and Reames, 2008).

The results of our studies demonstrate:

1. Analysis of the measurement with neutron monitors, the numerical simulation, and GOES spacecraft data indicates that the large proton events are generated in the current sheet during a flare. The protons from the western flares arrive to the Earth environment along the spiral magnetic lines. This proton flux arrives with the steep front. A clear correlation of small proton events with specific flares is not observed. Apparently, small events are generated by the flares on the back side of the Sun.

2. To accelerate protons by the shock waves it is necessary that the Larmor radius of the particles does not exceed the thickness of the shock front. This condition is impossible in the interplanetary plasma for particles with the energy order of 10 GeV. Such a mechanism could accelerate particles only to the certain energy in the strong magnetic field near the Sun.

3. The measurements of fast proton fluxes of solar cosmic rays reveal four characteristic time scales:

a). The typical duration of accelerated proton generation is equal to the duration of the flare  $t_{SF} \sim 20\div 30$  min.

b). A typical duration of the accelerated protons flux measured at the Earth orbit is equal to the propagation time of the solar wind from the Sun  $t_{SW} = 1AU/V_{SW} \sim 3$  days. This means that the most delayed protons have been captured by the magnetic field of the solar wind and transported to the Earth orbit across the magnetic field with the solar wind velocity.

c). The flux of protons from western flares increases very fast ( $\sim 20$  min). The delay of the steep flux front of fast protons generated by the western flare  $t_F = 15 - 20$  min is determined by the flight time of a proton to the Earth orbit along the magnetic lines of the Archimedes spiral  $\sim 1.5$  AU/c. Collisionless propagation of protons along helical magnetic field lines carries information about the spectrum of flare protons, which according to the neutron monitors data possess the exponential form.

d). The measured front of the proton flux from eastern flares is never being steep. The flux of relativistic protons from the eastern flare arrives to the Earth with the delay of  $t_D \sim 3 - 5$  hours. The flux increases very slow ( $\sim 1$  day). The long front can be associated with the diffusion across the magnetic field lines due to scattering of protons with magnetic inhomogeneities.

4. Proton events from eastern flares are recorded less frequently than the proton events from western flares. The absence of magnetic field lines connecting the eastern flares with Earth environment does not always allow proton flux to reach Earth environment.

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