# Plasma Density Fluctuations in Turbulent Flows of Active Regions at the Solar Photosphere

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***Abstract.*** Formation of small-scale fluctuations in plasma density of turbulent flows in active regions (plages) of the photosphere and dependence of their characteristics on the magnetic field strength B are considered in the present paper. The process is described in the framework of three-fluid approach. Taking into account a low degree of ionization of the gas in the photosphere, we assume that ion-electron plasma is embedded in the flow of gas and has no influence on its motion. We also assume that the statistics of random velocity field of gas corresponds to the Kolmogorov turbulence. For analysis of the effect of magnetic field on the plasma density fluctuations, analytic expressions describing their spatial spectrum and RMS level are derived. Estimations of the spectral shape and the fluctuation level are made for the photosphere near 300 km altitude under the strength B from 100 to 1000 G. It is shown that the RMS amplitude of fluctuations (with length-scales smaller than 100 km) around the mean plasma density has to increase from 8.23 to 8.79 % with the magnetic field strength. The spatial spectrum can be approximated by a power law and the power index has to increase from –1.57 (B=100 G) to –1.20 (B=1000 G).



**Keywords:** Sun, photosphere, plages, turbulence, plasma fluctuations

## Introduction

The solar surface is divided into active and quiet regions because of differences in the morphology of magnetic fields at photospheric level (Cattaneo, 1999). Individual active regions produced most of variability of the Sun on time-scales of weeks to months (Penza, Caccin, and Del Moro, 2004). It is also well established by observations that motions in the solar photosphere are in fully developed turbulent state (Fraubert-Scholl et al., 1995; Ruzmaikin et al., 1996; Cadavid et al., 1998; Lawrence et al., 2001; Bushby and Houghton, 2005).

In this report we consider formation of plasma density fluctuations in turbulent flows of photospheric plage regions. Plage is the part of active region outside sunspots which contains a mean magnetic field of a few hundred gauss. The plage regions are associated with strong photospheric downflows and last about an hour (Preist, 1984). Data of observations show that the properties of the photosphere differ qualitatively and quantitatively between the plage, where the magnetic field is 100 G or more, and its quiet surroundings (Title et al., 1992).

Plasma in the solar photosphere is only weakly ionized, with charged and neutral species coupled by collisions (Krinberg and Teplitzkaya, 1971; Fontenla et al., 1993). The charged components are a minor species and have no influence on both the average motion and turbulent pulsations of neutral gas (at least outside sunspots with strong magnetic fields) (Parker, 1979). It may be said that the ion-electron plasma of the photosphere is embedded in the turbulent flows of neutral gas. Such a turbulent situation may result in fluctuations of plasma density, even though the gas is incompressible.

Consideration of small-scale fluctuations seems to be interesting because of an essential progress in spatial resolution of observations of the solar photosphere (Berger et al. 2004; Bushby and Houghton, 2005; Ishikawa et al., 2008; De Wijn et al., 2009).

It should be noted that the fluctuations have to be important for observable fluctuations of spectral lines of ionized components in the photosphere and for appearance of random component of the local magnetic field there.

The aim of the paper is to consider formation of small-scale fluctuations in plasma density of turbulent flows in active regions (plages) of the solar photosphere and dependence of their characteristics on the magnetic field strength

## Basic assumptions and equations

To describe turbulent mixing in the solar photosphere (which is a slow process) a three-fluid model can be used: electrons, ions with a mean mass $m_i$ and neutral gas. Since the charged particles are passive contaminants, they have no influence on motions of the gas and the gas velocity field **u**(**x**, *t*) may be treated as a known function of position and time. The gas in the photosphere can be regarded as incompressible, $\nabla\mathbf{u}=0$. The behavior of charged particles embedded in the gas flow can be described by the following set of equations (Kyzyurov, 2007):

$$\partial N_s / \partial t + \nabla(N_s \mathrm{v}_s) = 0\,, \qquad (1)$$

$$\tau_s^{-1}(\mathbf{v}_s - \mathbf{u}) = q_s\mathbf{E}/m_s + \Omega_s(\mathbf{v}_s \times \mathbf{b}) - \mathrm{v}_{Ts}^2 N_s^{-1}\nabla N_s\,, \qquad (2)$$

where the variables are chosen as density Ns and velocity vs for each species (s≡i, e), qs is the particle charge (qe=–qi=–e), Ωs=qsB/msc is the gyrofrequency, τs is a characteristic time of charged particle collisions with neutrals, vTs is the thermal velocity, ms is the particle mass, b=B/B is the unit vector along the magnetic field B, E is the electric field.

In the photosphere Hall parameter for ions $\tau_i\Omega_i$<<1 and the assumptions of quasi-neutrality $N_e=N_i=N$ and isothermality $T_e=T_i=T_n=T$ are valid. Taking into account the only electric field that required to prevent charge separation (due to **E** electrons tend to follow ions, other electric fields are ignored), and eliminating the electric field from (2), we obtain the local drift velocity of ions (or plasma in general)

$$\mathbf{v}_i \approx \mathbf{u} + \tau_i\Omega_i(\mathbf{u}\times\mathbf{b}) - D_A N^{-1}\nabla N\,, \tag{3}$$

here DA is the ambipolar diffusion coefficient and is a scalar for small $\tau$i values (Krall & Trivelpiece 1973).

Using (3), the equation of continuity (1) then becomes

$$\partial N/\partial t + \nabla\cdot[N(\mathbf{u}+\tau_i\Omega_i(\mathbf{u}\times\mathbf{b}))] - D_A\nabla^2 N = 0\,. \tag{4}$$

The equation (4) describes dynamics of plasma density when plasma is embedded in a flow of neutral gas and relates plasma number density N with neutral gas velocity u.

In the case of turbulent flows the gas velocity may be separated into mean and fluctuating parts $\mathbf{u}=\mathbf{u_0}+\mathbf{u_1}$ ($\mathbf{u_0}$=<$\mathbf{u}$>, <$\mathbf{u_1}$>=0, $\mathbf{u_1} < \mathbf{u_0}$). The same may be made for plasma density $N=N_0+N_1$ ($N_0$=<$N$>, <$N_1$>=0, $N_1 < N_0$); $N_1$ represents plasma fluctuations around mean value, $N_0$, generated by the turbulent velocity field $\mathbf{u}_1$.

For length-scales of plasma fluctuations, ***l***, smaller than mean electron-density gradient, $L_N=(|\nabla N_0|/N_0)^{-1}$, we can write the following equation for relative fluctuations in plasma density, $\delta N=N_1/N_0$:

$$\partial\delta N/\partial t + \nabla\cdot(\delta N\mathbf{u}_1) - D_A\nabla^2\delta N = -(\mathbf{u}_1\mathbf{n})/L_N - \tau_i\Omega_i\mathbf{b}\cdot\nabla\times\mathbf{u}_1 \tag{5}$$

where $\mathbf{n}=L_N(\nabla N_0/N_0)$ is the unit vector along the mean plasma-density gradient.

The process of formation of plasma density fluctuations with scales smaller than the length-scale of mean plasma density gradient is described by (5). It may be seen that the first term on the RHS of (5) is more important at larger scales $l > \tau_i\Omega_i L_N$, and the second is more important for smaller ones $l < \tau_i\Omega_i L_N$. The process in which the neutral gas turbulence in conjunction with a background electron-density gradient produce plasma fluctuations by mixing regions of high and low density dominates at the larger scales, while the interaction of plasma imbedded in the turbulent motions of neutral gas with magnetic field is more important for generation of the small-scale fluctuations.

## Spectrum of plasma density fluctuations

Under the assumption that the random fields u1(x, t) and δN(x, t) are stationary function of x and t with Fourier transforms

$$u_{1j}(\mathbf{x},t)=\int \mathbf{dk}\,\mathbf{d\omega}\,u_{1j}(\mathbf{k},\omega)\exp(i\mathbf{kx}-i\omega t), \tag{6}$$

$$\delta N(\mathbf{x},t)=\int \mathbf{dk}\,\mathbf{d\omega}\,\delta N(\mathbf{k},\omega)\exp(i\mathbf{kx}-i\omega t), \tag{7}$$

the Fourier transform of (5) is then

$$(D_A k^2 - i\omega)\delta N(\mathbf{k},\omega) + ik_j\int dk'd\omega'\delta N(\mathbf{k}',\omega')u_{1j}(\mathbf{k}-\mathbf{k}',\omega-\omega') =$$
$$= -(\mathbf{n}\cdot\mathbf{u}_1(\mathbf{k},\omega))/L_N - i\tau_i\Omega_i\mathbf{k}(\mathbf{u}_1\times\mathbf{b})\,. \tag{8}$$

The convolution term on the LHS of (8) represents the contribution of mode interactions in the process of plasma fluctuation generation. If we take it into account through the coefficient of turbulent diffusion $K_T$, then

$$\delta N(\mathbf{k},\omega) = (-L_N^{-1}\mathbf{n} - i\tau_i\Omega_i(\mathbf{b}\times\mathbf{k}))[(D_A+K_T)k^2 - i\omega]^{-1}\mathbf{u}_1(\mathbf{k},\omega)\,. \tag{9}$$

Using known relations for statistically homogeneous and stationary random fields:

$$\left\langle u_{1\alpha}(\mathbf{k},\omega)\cdot u_{1\beta}^*(\mathbf{k}',\omega')\right\rangle = \Phi_{\alpha\beta}(\mathbf{k},\omega)\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\,, \tag{10}$$

$$\left\langle \delta N(\mathbf{k},\omega)\cdot\delta N^*(\mathbf{k}',\omega')\right\rangle = \Psi(\mathbf{k},\omega)\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\,, \tag{11}$$

where the * denotes a complex conjugate, and the spectrum tensor of turbulent velocity field $\Phi_{\alpha\beta}$ takes the form (Cadavid et al., 1998; Kyzyurov, 2007)

$$\Phi_{\alpha\beta}(\mathbf{k},\omega) = D_{\alpha\beta}(\mathbf{k})\tau_t(k)\,E(k)\,[4\pi^2(1+\omega^2\tau_t^2)]^{-1},\quad k_0<k<k_\nu \tag{12}$$

($D_{\alpha\beta}=\delta_{\alpha\beta}-k_\alpha k_\beta/k^2$ is the projection operator, $E(k)=C_1\varepsilon^{2/3}k^{-5/3}$ is the energy spectrum function, $\tau_t(k)=(\nu k^2+\varepsilon^{1/3}k^{2/3})^{-1}$ is the decay time of eddy with a length-scale $k^{-1}$, $k_0^{-1}$ is the basic energy input scale, $k_\nu=(\varepsilon/\nu^3)^{1/4}$ is the Kolmogorov dissipation wavenumber, ν is the kinematic viscosity of the gas, ε is the rate of turbulent energy dissipation per unit mass, the Kolmogorov constant $C_1$ is around 1.5 (McComb, 1995); at viscous length-scale $k_\nu^{-1}$, viscous dissipation is adequate to dissipate the energy at the rate ε) and taking (9) into account, we then obtain an expression for spatiotemporal spectrum of relative fluctuations in plasma density:

$$\Psi(\mathbf{k},\omega) = [4\pi^2(1+\omega^2\tau_k^2)(1+\omega^2\tau_t^2)]^{-1}\tau_t\tau_k^2 Q(\mathbf{k})\,, \tag{13}$$

here: $\tau_k=(D_Ak^2+K_Tk^2)^{-1}=(D_Ak^2+\varepsilon^{1/3}k^{2/3})^{-1}$,

$$Q(\mathbf{k}) = [(\mathbf{n}\times\mathbf{k})^2/(L_Nk)^2 + (\mathbf{b}\times\mathbf{k})^2/(\tau_i\Omega_i)^2]C_1\varepsilon^{2/3}k^{-11/3},$$

$$L_N^{-1}<k<k_d,$$

$k_d=(\varepsilon/D_A^3)^{1/4}$ is the Oboukhov-Corrsin wavenumber known in the theory of passive scalar turbulent convection (McComb, Filipiak, and Shanmugasundaram, 1992); in the present case the fluctuation length-scale at which $K_T=D_A$ is defined by $k_d^{-1}$.

From (13) one can obtain the spatial spectrum of δN

$$P_N(\mathbf{k}) = \int_{-\infty}^{\infty}\Psi(\mathbf{k},\omega)d\omega = [4\pi(1+\tau_t/\tau_k)]^{-1}\tau_t\tau_k Q(\mathbf{k})\,. \tag{14}$$

Using (14) a mean-square level of $\delta N$ in the range ($k_1$, $k_2$) may be calculated:

$$\left\langle \delta N^2 \right\rangle = \int P_N(\mathbf{k}) d\mathbf{k} = S((k_2/k_d)^{4/3}) - S((k_1/k_d)^{4/3}), \quad (15)$$

where

$$S(x) = \frac{3}{8} L_N^{-2} k_d^{-2} x^{-3/2} [(3+\mathrm{Pr}) - 2/3] +$$
$$\frac{3}{2} \frac{L_N^{-2} k_d^{-2}}{1-\mathrm{Pr}} [\operatorname{arctg} x^{1/2} - ((1+\mathrm{Pr})/2)^{5/2} \operatorname{arctg}(x(1+\mathrm{Pr})/2)^{1/2}] +$$
$$+\frac{3}{8} \frac{\tau_i^2 \Omega_i^2}{1-\mathrm{Pr}} [2\ln(x/(1+x)) - (1+\mathrm{Pr})\ln(x(1+\mathrm{Pr})/(2+x(1+\mathrm{Pr})))]$$

here **Pr**=$\nu/D_A$ is the diffusion Prandtl number.

The expression (14) represents the three-dimensional spectrum of plasma density fluctuations generated by turbulent motions of gas in a plage region of the photosphere, but in experiments one-dimensional spectra are usually measured. The 1D spectrum of plasma fluctuations that could be measured along an arbitrary **z**-direction may be obtained from (14):

$$P_N(k_z) = \int_0^{k_\zeta} k_\perp dk_\perp \int_0^{2\pi} P_N(\mathbf{k}) d\phi =$$

$$\frac{1}{4} \int_0^{k_\zeta} (L_N^{-2} f(k_\perp, k_z, \theta_1) + \tau_i^2 \Omega_i^2 k^2 f(k_\perp, k_z, \theta_2)) F(k) k^{-7} k_\perp dk_\perp, \quad (16)$$

where $f(k_\perp,k_z,\theta)=k_\perp^2+k_\perp^2\cos^2\theta+2k_z^2\sin^2\theta$, $\theta_1$ is the angle between **z** and **n**, $\theta_2$ between **z** and **b**, $k_\zeta^2=k_d^2-k_z^2$, $k^2= k_\perp^2+k_z^2$, $F(k)=[(1+(k/k_d)^{4/3})\times(2+(k/k_d)^{4/3}+(k/k_\nu)^{4/3})]^{-1}$

Expressions (15), (16) give an opportunity to estimate intensity and spectrum of the plasma fluctuations in plague regions of the photosphere.

## Plasma density fluctuations in plage regions of the photosphere

Data of observations show that magnetic field in plage regions may be directed both vertical and horizontal (Ishikawa et al., 2008; De Wijn et al., 2009), but the magnetic field is mostly vertical in the interior of plage (De Wijn et al., 2009).

In the case when n and b are in vertical direction, while the measurement direction z is horizontal then $\theta1=\theta2=\pi/2$ and (16) takes the form

$$P_N(k_z) = \frac{1}{4} \int_0^{k_\zeta} (L_N^{-2} + \tau_i^2 \Omega_i^2 k^2)(k_\perp^2 + 2k_z^2) F(k) k^{-7} k_\perp dk_\perp \quad (17)$$

The plasma fluctuations generated by neutral turbulence are analyzed near height of 300 km. The outer scale of turbulence is regarded to be equal to the size of cell with downflow, $k_0^{-1}=L_0\approx610$ km (Kostyk, 1985). We suppose that $L_N\approx L_0$. If on the length-scale $L_0$ the mean gas velocity is $\upsilon_0$, then $\varepsilon=\upsilon_0^3/L_0$. Parameters of the photosphere taken from (Krinberg, 1971; Kostyk, 1985; Fontenla et al. 1993) together with the calculated values needed for using (15), (17) are: $T$=5070 K, $N_n$=1.56×10$^{22}$ m$^{-3}$, $N_e$=2.10×10$^{18}$ m$^{-3}$, $m_i$ = 26.5 AMU, $\upsilon_0$ =1.48 km s$^{-1}$, $\tau_i^{-1}$=2.98×10$^7$ s$^{-1}$, $\nu$=1.74 m$^2$ s$^{-1}$, $D_A$= 0.11 m$^2$ s$^{-1}$, $\varepsilon$=5329 m$^2$ s$^{-1}$, $k_\nu^{-1}$= =17.73 cm, $k_d^{-1}$=2.18 cm. For existence of the neutral turbulence the Reynolds number must be large and in our case the condition is fulfilled Re=$\upsilon_0^3 L_0/\nu$=5.17×10$^8$. Values of the Hall parameter for ions in plage regions with $B$=100 G and $B$=1000 G are presented in Table as well as characteristics of plasma fluctuations calculated with use of (15), (17). The limits of integration in (15) are $k_1=1/L_m$, $k_2= k_\nu$ ($L_m$=100 km). The length-scale at which change in the slope of spectrum $P_N(k_z)$ takes place ($L_B=\tau_i\Omega_i L_N$) is also shown in Table. The wavenumber corresponding to the length-scale $L_B$ defines two range of wavenumbers: for excitation of the fluctuations with $k_z<L_B^{-1}$ the gradient in mean plasma density is more important while the role of magnetic field is prevailing at $k_z>L_B^{-1}$. The value $\gamma$ is a power index when the spectrum is approximated by a simple power law $k_z^\gamma$. Figure shows spectrum $P_N(k_z)$ calculated using (17) for the plage region at altitude $h$=300 km with $B$=100 G – line 1, and line 2 – for the plage region with $B$=1000 G; a straight line represents the power law $k_z^{-5/3}$.

Table Calculated values of the Hall parameter and characteristics of the plasma fluctuations in plage regions of the photosphere near altitude $h$=300 km

| B, G | $\tau_i\Omega_i$ | $L_B=\tau_i\Omega_i L_N$, m | $\langle\delta N^2\rangle^{1/2}$, % | $\gamma$ |
|---|---|---|---|---|
| 100 | 1.21.10–3 | 737.6 | 8.23 | –1.57 |
| 1000 | 1.21×10–2 | 7375.9 | 8.79 | –1.20 |

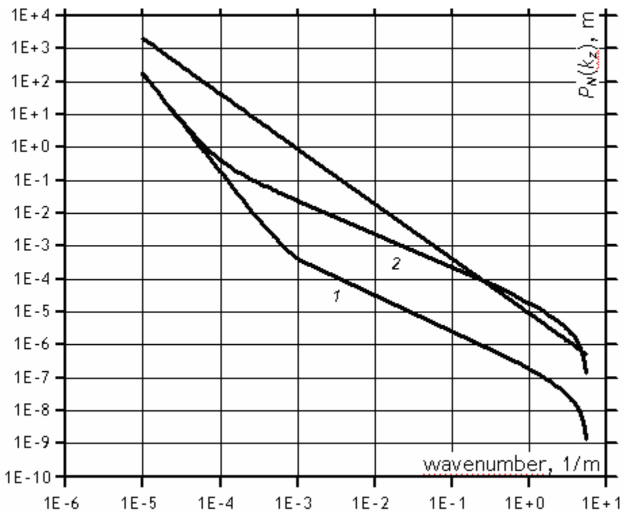


Fig. Spectrum $P_N(k_z)$ calculated using (17) for the plage region at altitude h=300 km when $B$=100 G - line 1, and for $B$=1000 G - line 2; a straight line represents the power law $k_z^{-5/3}$

## Conclusions

An analytic expression (17) for the 1D spectrum of the plasma density fluctuations (with length-scales corresponding to the inertial range of turbulence) in turbulent flows of photospheric gas in plage regions as well as the formula (15) for estimation of the RMS fluctuation level were obtained in this work.

The expression (17) allowed us to estimate changes in the shape of horizontal fluctuation spectrum at altitude $h$=300 km under increase in the magnetic field strength $B$ from 100 to 1000 Gauss (see Figure). We considered the case of the magnetic field parallel to the vertical gradient in mean plasma density. When the spectrum was approximated by a simple power law $k_z^{\gamma}$, then the power index $\gamma$ increased from −1.57 to −1.20 with increasing $B$. The change in the shape of the fluctuation spectrum $P_N(k_z)$ with $B$ is connected with the growth of magnetic field role in generation of the plasma density fluctuations due to widening of the range in which they are formed by interaction of the plasma imbedded in turbulent motions of neutral gas with the magnetic field.

We also calculated the RMS level of the fluctuations $\langle \delta N^2 \rangle^{1/2}$. The increase in $B$ resulted in the rise of the level from 8.23 to 8.79% relative to the local mean plasma density. Relatively weak influence of B on the fluctuation amplitude is explained by a more important role of local gradient in the mean plasma density for the fluctuation generation. Destruction of the gradient by turbulent mixing of the gas dominates in the formation of fluctuations with smaller wavenumbers, while interaction of plasma submerged in the turbulent gas flow becomes more important for larger kz. It is known that velocity fluctuations with smaller wavenumbers (larger length-scales) contain more turbulent energy than ones with larger kz.

The considered plasma fluctuations seem to be important for better understanding of observable fluctuations of spectral lines of ionized components in the photosphere as well as for explanation of appearance of random component in the local magnetic field there. Due to turbulent plasma (electron) density fluctuations, random component in plasma conductivity has to appear that results in the random component of magnetic field (Parker, 1979).

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